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DHC-SP-TN 164

TEMPERATURE GRADIENTS AND PROFILE CHANGES IN LONG TUBULAR ELEMENTS DUE TO INCIDENT RADIATION

PREPARED FOR

NAVAL RESEARCH LABORATORIES
Contract No: NONR 3592(00)(X)

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December 1962

STUDIES CARRIED OUT FOR
NAVAL RESEARCH LABORATORIES
under
CONTRACT NO: NONR 3592(00)(X)
December 1962

ERRATA PAGE TO DHC-SP-TN 164

Page iv	paragraph 3, second line:- replace "minimum" by "maximum"
Page 6	equation (8) shall be written as:

$$\theta_{(x)} = \left(\frac{\pi}{2} - \gamma_{(x)} N \right) \frac{180}{\pi}$$

Figure 9	graph No. 41 (10.1.63) shall be replaced by graph No. 41A (March 5, 1963)
Page 35	paragraph 2, second line:- Figure 12 shall read Figure 13

Two sets of equations in Part II have been numbered 15 to 22, one set on pages 21 to 23 and the other set on pages 24 to 27.

March 6, 1963

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PART II

MATHEMATICAL ANALYSIS

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SUMMARY

The temperature difference between the bright and dark side of the specified 800 - ft., half inch diameter, Beryllium Copper element, when exposed to sun radiation, will reach 12°F. This difference will cause the element to bend away from the radiation source in an asymptotic form. The tip deflection will be about 550 ft., with a minimum radius of curvature of 333 ft.

The location of the point of minimum radius of curvature can be anywhere along the axis of the element from the root to the tip, depending on the element's initial direction with respect to the radiation source.

The point of minimum radius of curvature coincides with the point of minimum element temperature, which will be about 850°R with sun and earth radiation, and 785° R in outer space.

The temperature difference along the cross section, and the resulting profile change, are determined by the material and its surface absorptivity only and are independent of the emissivity; while the maximum temperature depends on the ratio of absorptivity to emissivity only.

The magnitude of the time constant of the specified element with respect to step changes in radiation is about 3 minutes. The final profile (96%) will be established in about 7 minutes after a sudden extension of the element into the radiation field.

PART I

EVALUATION AND DISCUSSION OF ANALYTICAL RESULTS

PART I

EVALUATION AND DISCUSSION OF ANALYTICAL RESULTS

1.0 INTRODUCTION

This study of the effect of radiant energy impinging on long tubular elements in space investigates the temperature distribution along the perimeter and the axis of the element and the resulting profile changes. The results are condensed into equations and into a Profile Chart, allowing examination of the particular profile of an element in outer space, or in an orbit around the earth. The time constant of the element is also investigated analytically.

1.1 Specification of the Element

The specified element is considered as a closed tube for most of the investigations. A correction factor is discussed and proposed for an open tube with 180° overlap. Numerical values of temperatures along the perimeter and the axis, and values for the minimum radii of curvature and tip deflections are calculated for an element with the following specifications:

Element Material	Beryllium copper
Element Diameter	$d = 0.04167$ feet (0.50 inches)
Element Length	$l = 800$ feet
Wall Thickness	$t = 0.000167$ feet (0.002 inches)
Material Density	$\rho = 500$ lb/ft^3
Thermal Conductivity	$k = 44$ BTU/ft - $^{\circ}$F - hr
Thermal Expansion Coefficient	$e = 1.04 \times 10^{-5}$
Absorptivity Ratio	$\alpha = .45$
Emissivity Ratio	$\epsilon = .10$
Specific Heat	$c = .10$ BTU/lb - $^{\circ}$F
Overlap Factor	$\varphi = 1.5$

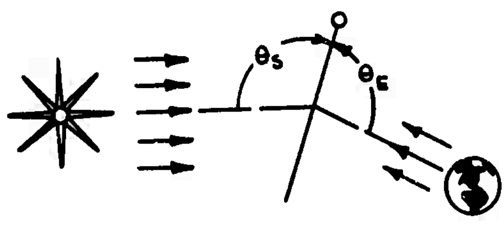
1.2 Specification of the Radiant Energy

Sun and earth radiation, and reflection of sun radiation by the earth and the atmosphere, are taken into account. These are assumed to have the following intensities : -

Sun Radiation	J_S	= 450 BTU/HR FT^2
Earth Radiation	J_E	= 70 BTU/HR FT^2
Earth Reflection	J_R	= 160 BTU/HR FT^2

Sun radiation is considered to be constant in the scope of this analysis. Earth radiation and reflection are subject to correction for altitude; the values applied, referring to the upper layers of the mesosphere, which is assumed to be 500 miles above the surface of the earth.

The amount of heat entering the element is also a function of the incident angle. This angle is defined in the analysis as the angle between the radiation direction and the axis of the element as seen from the root of the element.(See diagram below.)



2.0 THE ELEMENT IN OUTER SPACE

2.1 The Maximum Temperature Difference Along the Element Perimeter

The maximum temperature difference between the bright and dark side of the element occurs when the incident angle $\theta_S = 90°$. It is obtained by the equation:

$$\Delta T_S = \frac{d^2}{4tk} \alpha_S J_S \sin \theta_S \qquad (1)$$

It is independent of the emmissivity ratio ε. $T_{s\,(max)}$ is 12°F for the specified element.

2.2 The Temperature Profile Along the Element Perimeter

The Temperature Profile is shown in Figure 2. It is symmetrical on both sides of the diameter from the bright side to the dark. It is obtained from the equations:

$$\Delta T_S(1) = \Delta T_{S\,max}\left[\frac{1}{2\pi}\left(\frac{X}{r}\right)^2 + \cos\frac{X}{r} - .45\right] \qquad (2)$$

$$\Delta T_S(2) = \Delta T_{S\,max}\left[\frac{1}{2\pi}\left(\pi - \frac{X}{r}\right)^2 - .45\right] \qquad (3)$$

Equation 2 gives the profile in the first and fourth quadrants, and equation 3 the profile in the second and third quadrants. In both cases, X is a point on the perimeter expressed in terms of $r \times \pi$

2.3 Element Profile Change

The temperature difference between the bright and dark sides of the element will cause the element to bend away from the radiation source. The profile of the bent element, the tip deflection, and the radii of curvature along the element axis, are determined by a "Profile Number" which is defined as

Profile Number: $N_S = \frac{e}{d}\Delta T_s(max) = \frac{1}{4}\frac{de}{tk} J_s \alpha_s$ (4)

It has a value of 3×10^{-3} for the specified element.

Using this profile number, the element shape can be derived by the equation:

$$X = \frac{1}{N_S}\ln\cos(Y \times N_S) \qquad (5)$$

This profile has been plotted in Figure 2 for the specified element and a radiation incident angle $\theta_S = 90^\circ$. The X and Y axes are in feet; and the length of the element has been traced on the plotted profile. Figure 3 shows the same element for a radiation incident angle $\theta_S = 135^\circ$. Figure 4 shows the profiles of a dipole element with incident angles from 0 to 180°.

2.4 Explanation of the Profile Chart

The profile curves of Figures 2, 3 and 4 are extracted from the ProfileChart shown in Figure 5. Any 800 foot length traced on the curve

with the Profile Number 3×10^{-3}, can be a profile of the specified element. The root of the element is located at the intersection point of the incident angle line with the profile curve, and the tip of the element is found by tracing the length of the element (800 feet) clockwise on the profile curve.

The Profile Chart and equation 4 for the Profile Number, imply that elements with different design parameters can have the same profile change as long as the design parameters in equation 4 yield the same Profile Number. Hence one can examine the change of profile for a change of a particular design parameter, by comparing the profile curves with the appropriate numbers.

2.5 Several Elements on One Satellite

As long as the design ensures that the incident radiation angle is the same for all the elements, several elements on one satellite will have the same profile. A crossed dipole of elements as specified, will have a configuration as shown in Figure 6. The configuration will be maintained in the orbit around the sun as shown in Figure 7. The elements have the tendency to align themselves and the satellite radially with respect to the orbit, so that the same side of the satellite surface always faces the sun. The backward-bent elements which are exposed to the sun's pressure, will create a correcting torque as soon as the satellite deviates from the equilibrium either in the pitch or yaw axis. The dynamic properties of the elements; their damping effects, and time lags, have to be included in the orientation and stabilization system of the satellite.

2.6 Tip Deflection

The value X in Equation 5 represents the tip deflection from the initial straight profile. It can be obtained only by tracing the profile. However, for very small Profile Numbers, or for short elements, Y can be replaced by the element length: ℓ and Equation 5 can be simplified to:

$$x = \frac{1}{2} \ell^2 N \tag{6}$$

Thus the deflection for a 100 foot length would be 15 ft., when exposed to sun radiation only.

2.7 Minimum Radius of Curvature

The greater the temperature difference on the cross section, the more the element will bend at that section, and the smaller will be the radius of the curvature. The location of the minimum radius of curvature

will be at the point where the incident angle is 90°. In Figure 2 the location is at the root of the element and in Figure 3 where the tangent on the element is normal to the radiation direction. The radius at any point is given by

$$R_{(x)} = \frac{1}{N} \sin \theta_{(x)} \tag{7}$$

where $\theta_{(x)}$ is the angle between the tangent and the initial direction. The minimum radius of curvature for the specified element is 333 feet., when exposed to sun radiation. Angle $\theta_{(x)}$ can be obtained from the graph or computed from the equation:

$$\theta_{(x)} = \frac{\pi}{2} - Y_{(x)} N \tag{8}$$

2.8 Maximum Element Temperature

The element temperature depends on the absorptivity and emissivity ratios, and the incident radiation angle, and is otherwise independent of the element design parameters. At any point X along the axis the average element temperature is given by

$$T_{(x)} = \left(\frac{1}{\varepsilon \sigma \pi} \alpha J_s \sin \theta_{(x)} \right)^{1/4} \tag{9}$$

where $\theta_{(x)}$ is again the angle between the tangent on the element and the radiation direction. The location of the point of maximum temperature coincides with the location of the point of minimum radius of curvature. Figure 8 shows maximum element temperature versus the ratio α/ε, and Figure 9 depicts the temperature along the axis of one element in two different positions with respect to the sun. The maximum temperature of the specified element is 785°R when exposed to sun radiation only.

2.9 Element Response Time

It has been assumed that the element will have found structural and thermal equilibrium when the region of highest temperature has been reached. For this condition, the time constant $C\tau$, which is defined as the time in minutes necessary to reach 63.2% of maximum temperature rise after a small step increase in radiant energy, is: -

$$C_{\tau} = \frac{\varphi}{4} \times \frac{ct\rho}{\sigma\varepsilon} \times \frac{1}{T^{3}_{s\,max}} \qquad (10)$$

"C " for the specified element would be 3 minutes.

However, the actual temperature response curve for a large step increase involving sudden extension of the element into the sunlight, must be derived from the equation:

$$\Delta T_{(step)} = \frac{\sigma\varepsilon}{ct\rho\varphi}\left(T^{4}_{s\,max} - T^{4}\right)\Delta\tau\,(step) \qquad (11)$$

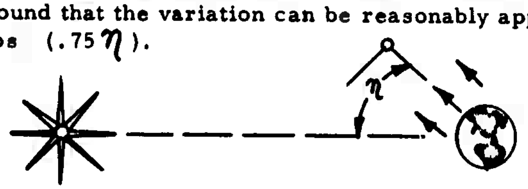
The transient temperature profile is shown for the specified element in Figure 10. The element needs 7 minutes to reach 96% of the final profile.

3.0 THE ELEMENT EXPOSED TO RADIANT ENERGY FROM SUN AND EARTH

3.1 Analysis and Definition of Earth Radiant Energy

A satellite in orbit around the Earth will be equally exposed to earth radiation due to the earth temperature at all positions of its orbit. The earth radiation constant has to be corrected for the distance of the satellite from the earth.

Reflected radiant energy can reach the satellite only as long as the sun and earth are in opposition with respect to the satellite. However, since the reflected sunlight is diffused in the atmosphere, it has been assumed that the reflection approaches zero, when the angle η between sun, Earth and Satellite reaches 120°. (See diagram below.) The variation of reflected radiation with η, between 0 and 120 degrees, can be represented by a complicated expression derived in Reference 1. However, it has been found that the variation can be reasonably approximated by the function cos $(.75\,\eta)$.



Both radiant energy sources - earth radiation and earth reflection - are subject to correction for altitude with the factor

$$F_A = \left(\frac{R + 500}{R + A} \right)^2 \qquad (12)$$

since radiation intensity varies inversely with the square of the distance. R is the radius of the earth, and A is the altitude of the satellite above the earth surface.

The effect of radiant energy from the earth, also depends on the magnitude of the angle between the sun and earth radiation with respect to the element cross section. This angle is illustrated in the diagram below and should not be confused with the angle η , although both may have the same value in particular positions.

3.2 The Maximum Temperature Difference Along the Perimeter

The maximum temperature difference and the profile along the perimeter is obtained by multiplying the appropriate equations 1 to 3 by the factor B to get:

$$\Delta T_{SRE} = \frac{d^2}{4tk} \alpha_S J_S \sin \theta_S \times B \qquad (13)$$

wherein B represents the effect of earth reflection of sunlight and earth radiation. B is developed as follows: the total amount of impringing radiation which will cause the temperature rise consists of:

Sun radiation:	$\alpha J_S \sin \theta_S$
Earth reflection:	$\alpha J_R \cos .75\eta \times \sin \theta_E \times F_A \cos^2 \delta$
Earth radiation:	$\varepsilon J_E \sin \theta_E \times F_A \cos^2 \delta$

The angles θ_S and θ_E were defined in 1.2 and the angle $.75\eta$ as well as the factor F_A in 3.1. Thus the total amount of impringing radiation is: -

$$\sum J = \alpha J_S \sin\theta_S \pm \left(\alpha J_R \cos .75\eta + \epsilon J_E\right) \sin\theta_E \times F_A \cos^2\delta \quad (14)$$

$$\sum J = \alpha J_S \sin\theta_S \left[1 \pm \left(\frac{J_R}{J_S} \cos .75\eta + \frac{J_E}{J_S}\frac{\epsilon}{\alpha}\right)\frac{\sin\theta_E}{\sin\theta_S} F_A \cos^2\delta\right] \quad (15)$$

where the sign + applies for $\delta < \pi/2$ and the sign $-$ for $\delta > \pi/2$
The term in brackets is called B thus:

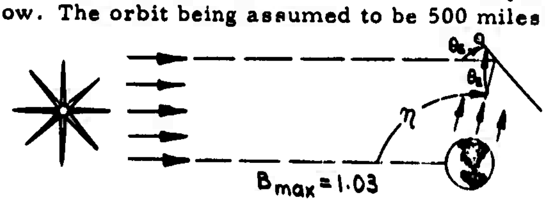
$$B_1 = 1 \pm \left(.355 \cos .75\eta + .155\frac{\epsilon}{\alpha}\right)\frac{\sin\theta_E}{\sin\theta_S} F_A \cos^2\delta \quad (16)$$

and when reflection approaches zero with $\eta > 120°$

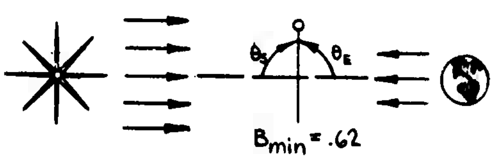
$$B_2 = 1 \pm .155\frac{\epsilon}{\alpha}\frac{\sin\theta_E}{\sin\theta_S} F_A \cos^2\delta \quad (17)$$

Inspection of the terms for B reveals that the magnitude of B depends mainly on the magnitude of the four angles which define the position of the element with respect to the sun and earth. This is because the ratio ϵ/α is very small, and is constant for the specified element.

The extreme values for B occur in the positions in the diagram below. The orbit being assumed to be 500 miles above the earth's surface.



$\theta_S = 90°$
$\theta_E = 165°$
$\eta = 105°$
$\delta = 0°$



$\theta_S = \theta_E = 90°$
$\eta = 0°$
$\delta = 180°$

3.3 The Temperature Profile Along the Perimeter

The temperature profiles for the above defined positions are shown in Figure 11. They represent the borderlines for all possible conditions. They are computed from the equation:

$$\Delta T_{ser} = \Delta T_s \times B \qquad (18)$$

Since B_{min} will increase, and B_{max} will decrease with orbits at higher altitudes, both will approach Unit 1 at an altitude of about 20,000 miles. This means that the influence of earth radiation and reflection will become virtually negligible at this altitude and above. The temperature profile then will be that indicated by the dotted line. This is the same as shown in Fig. 1.

3.4 Effect on Earth Radiant Energy on the Element Profile

The Profile Number of the element will be changed in the same ratio as the temperature gradient along the perimeter is changed. Thus equation 4 becomes:

$$N_{ser} = N_s \times B \qquad (19)$$

Because B_{min} was found to be .62 for the specified element with α_s/ϵ = .22 the minimum Profile Number for the specified element when exposed to sun and earth radiant energy will be:

$$N_{ser(min)} = 1.86 \times 10^{-3}$$

The appropriate profile for this Profile Number is marked in Figure 12 on the Profile Chart, for comparison with the profile for the number 3×10^{-3}. It is the lowest Profile Number which can occur in the most favourable position with sun and earth in opposition, with respect to the satellite. For all other positions particularly for all higher altitudes the Profile Number will increase until it approaches the value 3×10^{-3} at an altitude of about 20,000 ft.

3.5 The Elements in Orbits Around the Earth

Unless the satellite travels across the shadow of the earth, where the elements will have almost a straight profile - the elements have the tendency to align themselves to the direction of the sun radiation. This

applies for all orbits, polar orbits as well as for equatorial or semi-equatorial orbits.

In a polar orbit (Figure 13), with the satellite orientated to the earth so that always the same surface will face the earth, the elements will keep their configuration with respect to the earth and to the sun. They will not influence the orientation and stabilizing system of the satellite.

In a semi-equatorial orbit however as shown in Figure 14 the elements will try to align themselves and the satellite to the sun radiation, so the satellite will perform one revolution around its axis while performing one orbit cycle. Thus when the satellite would have an orientation control to face the earth always with the same side of the surface the orientation control has to cater for the disturbance caused by the elements.

In such cases the elements would take profiles as shown in Figure 15 and 16. In Figure 15 only one element is assumed, pointing to the earth in its initial position. For Figure 16 a dipole of two elements is assumed in the plane of the orbit, as tangents on the orbit path in their initial position. This configuration assumes that the orientation control keeps the satellite always in the position facing the earth with the same side of the surface.

3.6 Maximum Element Temperature Along the Element Axis

The temperature of the element on the point of Minimum Radius of Curvature is a function of the total radiant energy impinging on the element. Thus

$J_S \alpha \sin \theta_S$ in equation 9 is replaced by:

$$\sum J = J_S \alpha \sin \theta_S + (J_R \alpha \cos .75\eta + J_E \varepsilon) F_A \sin \theta_E \cos^2 \delta$$

which yields

$$T_{ser} = T_S \times B^{1/4} \qquad (20)$$

but with always the sign + after 1. (Equation 16 in 3.2.)

Thus the maximum temperature occurs for

$$F_A = 1 \text{ and } \theta_S = \theta_E = 90° \text{ and } \eta = 0° \text{ and } \delta = 180°,$$

when sun and earth are in opposition with respect to the satellite.

In this case the maximum temperature for the specified element is:

$$T_{ser(max.)} = 850°R$$

3.7 Element Response Time

Replacing $T_{s\ max}$ in equation 10 by $T_{ser} = T_{s\ max}\ B^{-3/4}$ yields the response time of the element when exposed to sun and earth radiant energy simultaneously. It would be about 2.35 minutes instead of about 3 minutes so that the overall pattern of the transient curve in this case will be similar to the curve as shown in Figure 10.

4.0 DISCUSSIONS OF BASIC ASSUMPTIONS

4.1 Average Temperature and Internal Radiation

The derivation of equation 1 to 3 assume an average element temperature along the perimeter of the tube for element surface radiation into space. More rigorous analysis has shown that for the specified element, the computed numerical values can be regarded as 2% higher than the true values. It was also found that inclusion of the internal radiation between the inside surface of the element would decrease the computed values in the same order. Both assumptions suggest, therefore, the introduction of a correction factor of 0.96 for average temperature.

4.2 Neglecting the Overlap

A closed tube of equal wall thickness has been assumed for this analysis. (i.e. - equal conductivity along the cross section.) The overlapped tube however has cross sections of varying thickness and undetermined thermal content. Difficulties arise when the overlap is included in the equations, due to the fact that the overlapped sections can have multiple positions with respect to the radiation source. They will change their position continuously, due to twisting effect of very long elements.

The evaluation of four distinct configurations suggest that a 180° overlap will decrease the temperature gradient in the order of 25%, calling for a further correction factor of 0.75.

4.3 Suggestion of a Tolerance Band for the Results

If the correction factors are included, all the computed numerical values of this analysis, would be about 30% to high. However, a number of other assumptions may call for corrections in the other direction. For instance, all the assumed numerical values of the absorptivity and emissivity ratios, as well as the values of the thermal conductivity, and the thermal expansion coefficient, may be larger or smaller than assumed. In view of this it is felt that the actual profiles will be within a $\pm$ 30% band about the computed values. This tolerance can be significantly reduced when α and ε values for the material are more accurately determined.

TEMPERATURE PROFILE ALONG THE PERIMETER OF A TUBULAR ELEMENT IN OUTER SPACE ELEMENT PROFILE: $N=3\times10^{-3}$	GRAPH NO.: 35	
	DRAWN: AG.	DATE: 10-1-63
	APPR:	

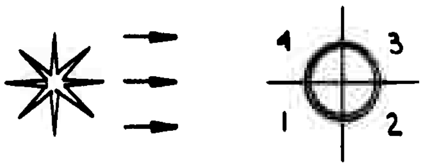
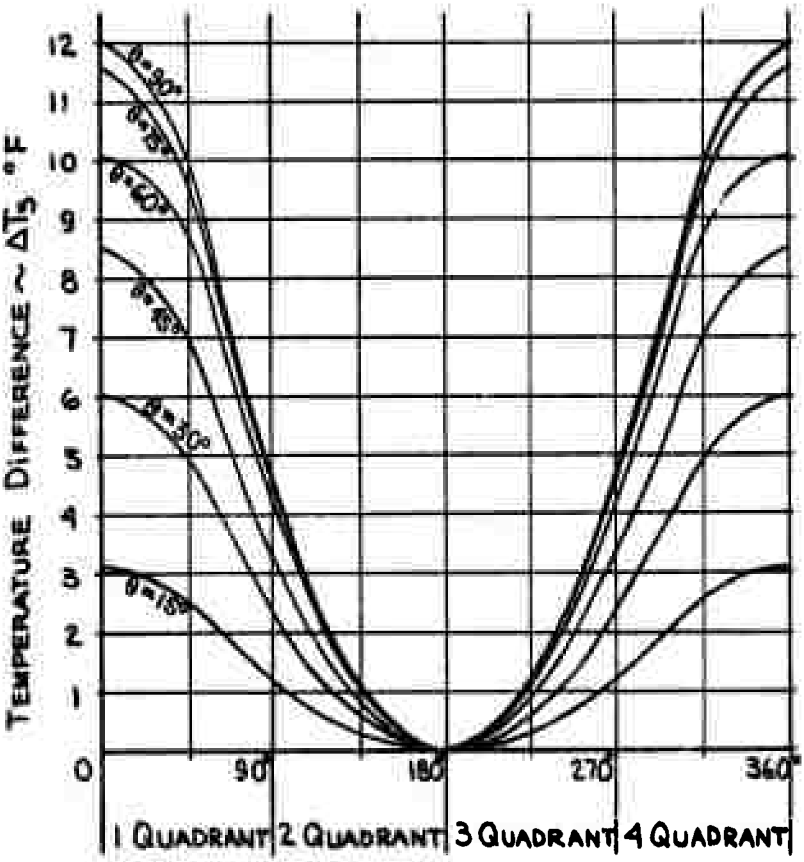


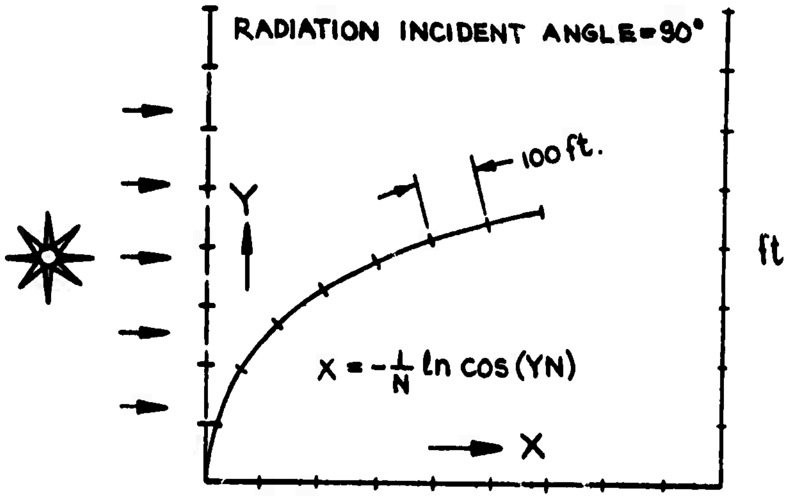
Fig. 1



$$\Delta T_S = \frac{1}{4}\frac{d^2}{tk} J_S \alpha_S \sin\theta_S$$

DEFLECTION PROFILES OF A TUBULAR ELEMENT IN OUTER SPACE ELEMENT PROFILE: N = 3×10⁻³ 800 FT. LONG	GRAPH NO.: 36	
	DRAWN: AW	DATE: 10-1-63
	APPR.:	

Fig. 2



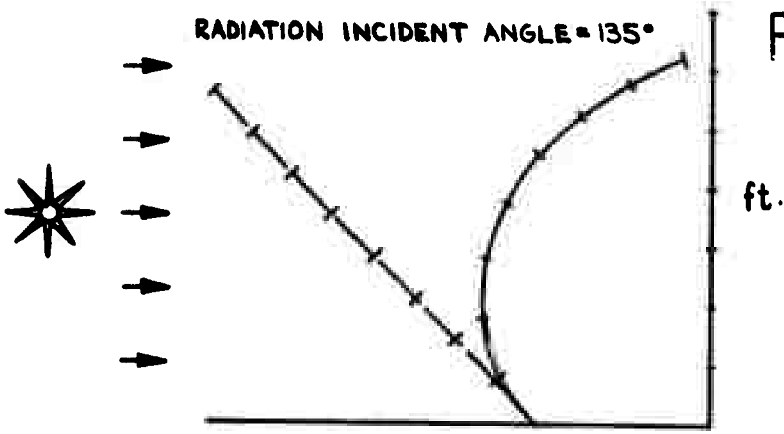
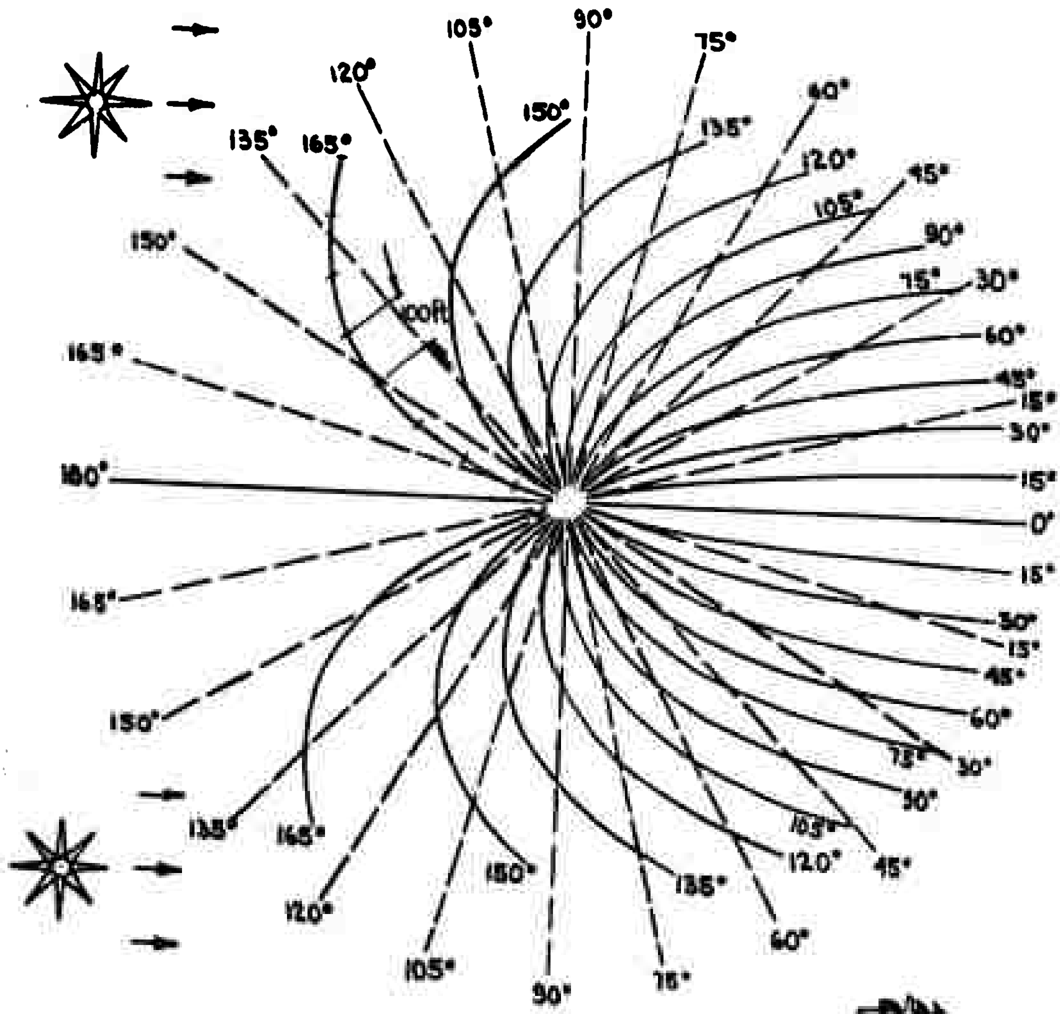


Fig. 3

DEFLECTION PROFILES OF A TUBULAR ELEMENT IN OUTER SPACE FOR VARIOUS RADIATION INCIDENT ANGLES ELEMENT PROFILE : $N = 3 \times 10^{-3}$; 800 FT. LONG	GRAPH NO.: 37	
	DRAWN:	DATE: 10-1-63
	APPR.:	

Fig. 4



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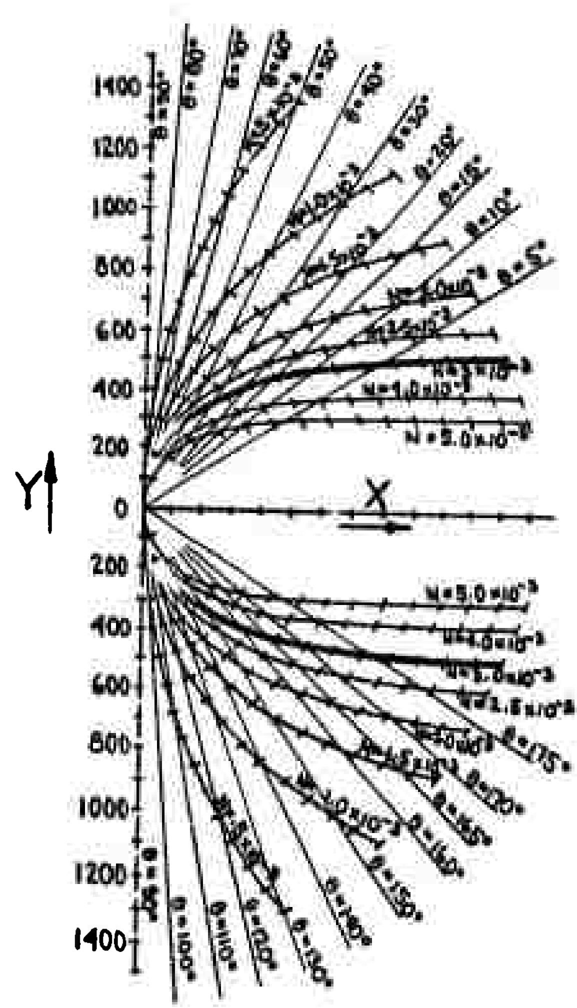
DEFLECTION PROFILE CHART	GRAPH NO. : 38	
	DRAWN: AV	DATE: 10-1-63
	APPR.:	

$$X = -\frac{1}{N} \ln \cos(YN)$$

Fig. 5



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DEFLECTION PROFILES OF TUBULAR ELEMENTS IN OUTER SPACE ELEMENT PROFILE: $N = 3 \times 10^{-3}$; 800 FT. LONG	GRAPH NO.: 39	
	DRAWN: AW.	DATE: 11-1-63
	APPR.:	

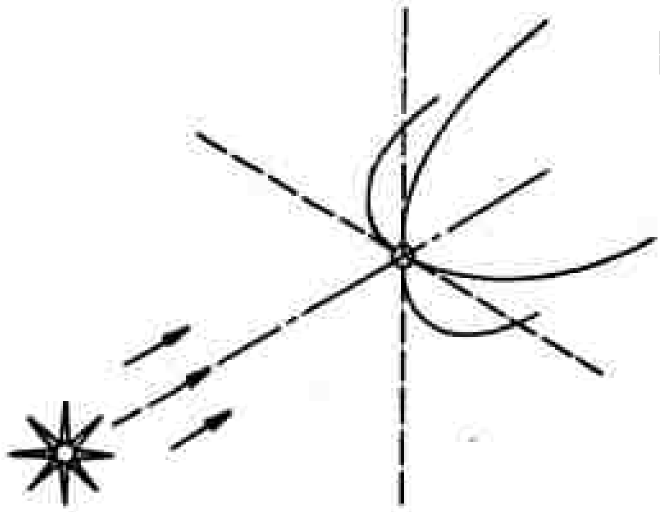

Fig. 6

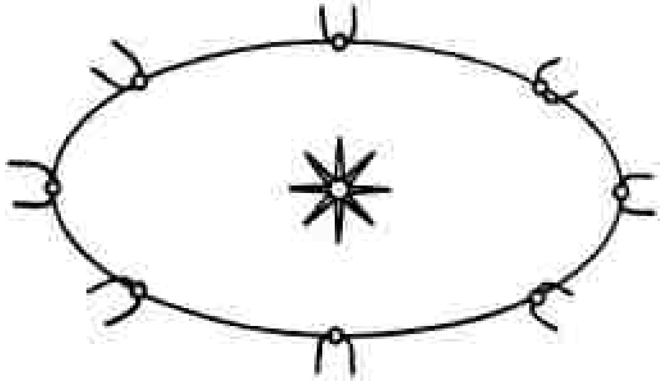

Fig. 7

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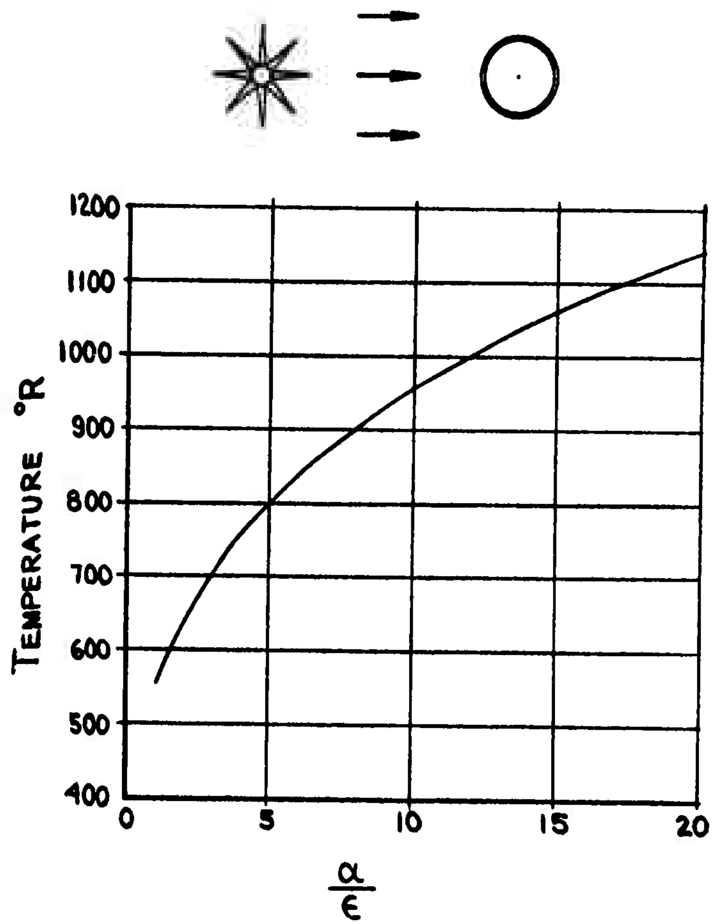
MAXIMUM TEMPERATURE OF A TUBULAR ELEMENT IN OUTER SPACE	GRAPH NO.: 40	
	DRAWN: AW.	DATE: 10-1-63
	APPR.:	

$$T_m = \left(\frac{\alpha_s J_s}{\sigma \epsilon_s \pi} \right)^{1/4}$$

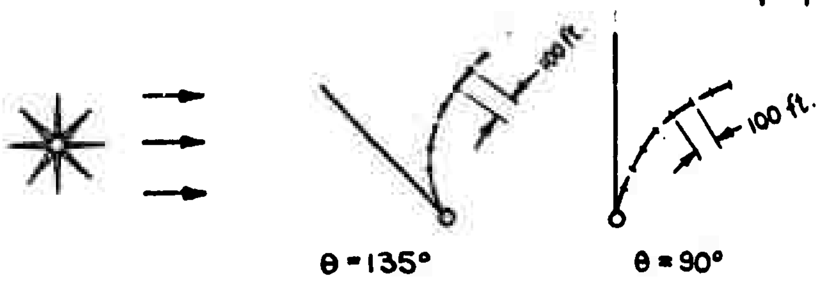
Fig. 8

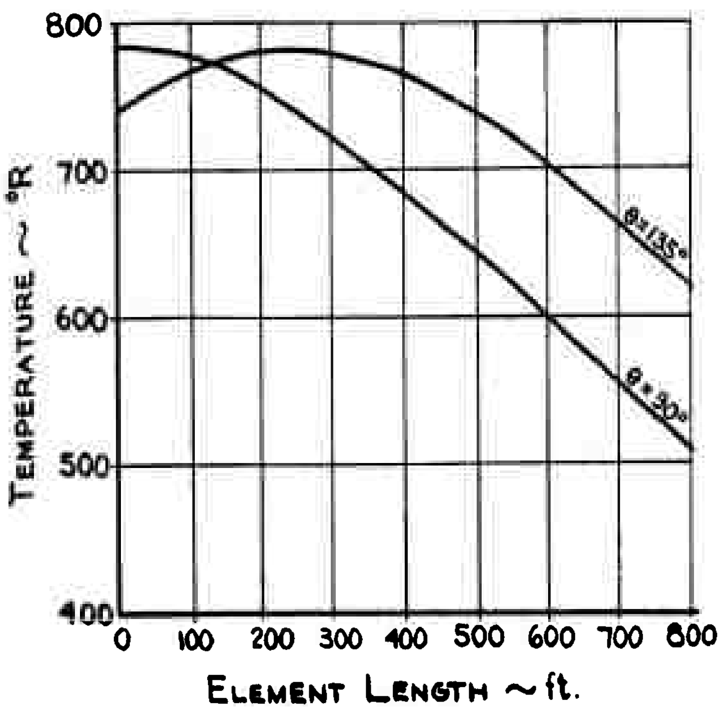


REPLACING GRAPH NO. 41/10-1-63

TEMPERATURE ALONG THE AXIS OF A TUBULAR ELEMENT IN OUTER SPACE ELEMENT PROFILE: $N = 3 \times 10^{-3}$	GRAPH NO.: 41A	
	DRAWN: A.V.	DATE: 6-3-63
	APPR.: [illegible]	

Fig. 9





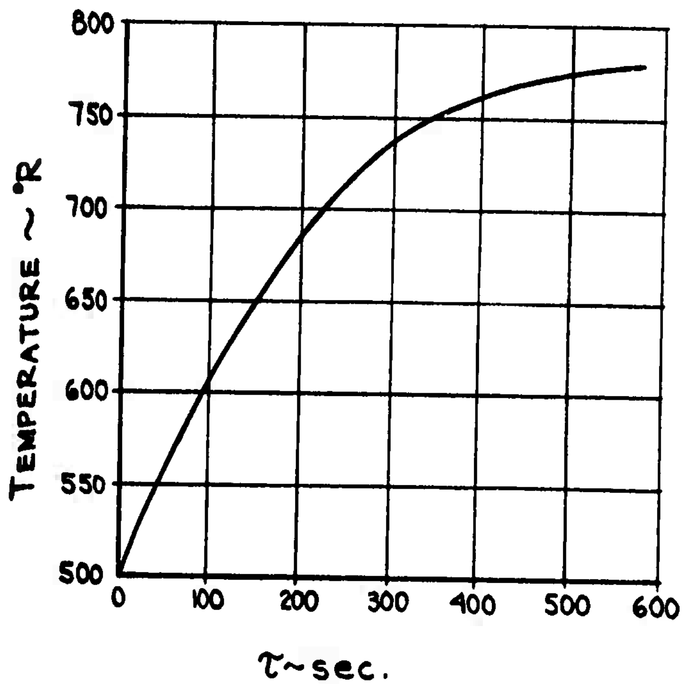
TRANSIENT TEMPERATURE OF A TUBULAR ELEMENT IN OUTER SPACE DUE TO A STEP CHANGE IN VARIATION FROM 0 TO 100%	GRAPH NO.: 42	
	DRAWN: AW.	DATE: 11-1-63
	APPR.:	

OVERLAP = 180°
C_p = .1 Btu./lb.-°F.
ρ = 495 lbs/ft³

t = .002 in.
ϵ = .1
φ = 1.5

Fig. 10

$$\Delta T = \frac{\sigma \epsilon}{\varphi \rho C_p t}\left(T_{max}^4 - T^4\right)\Delta \tau$$



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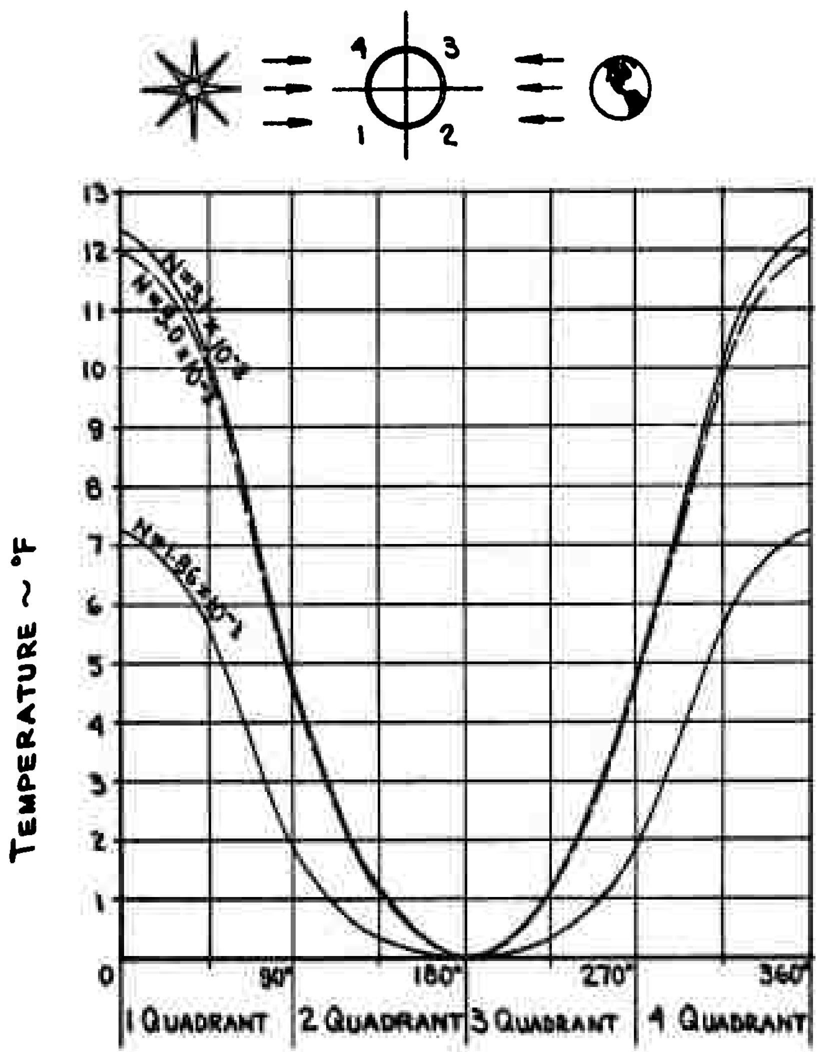
TEMPERATURE PROFILE ALONG THE PERIMETER OF A TUBULAR ELEMENT IN ORBITS AROUND THE EARTH	GRAPH NO.: 43	
ELEMENT PROFILE : $N = 1.86 \times 10^{-3}$ TO 3.1×10^{-3}	DRAWN: *ar*	DATE 11-1-63
	APPR.:	

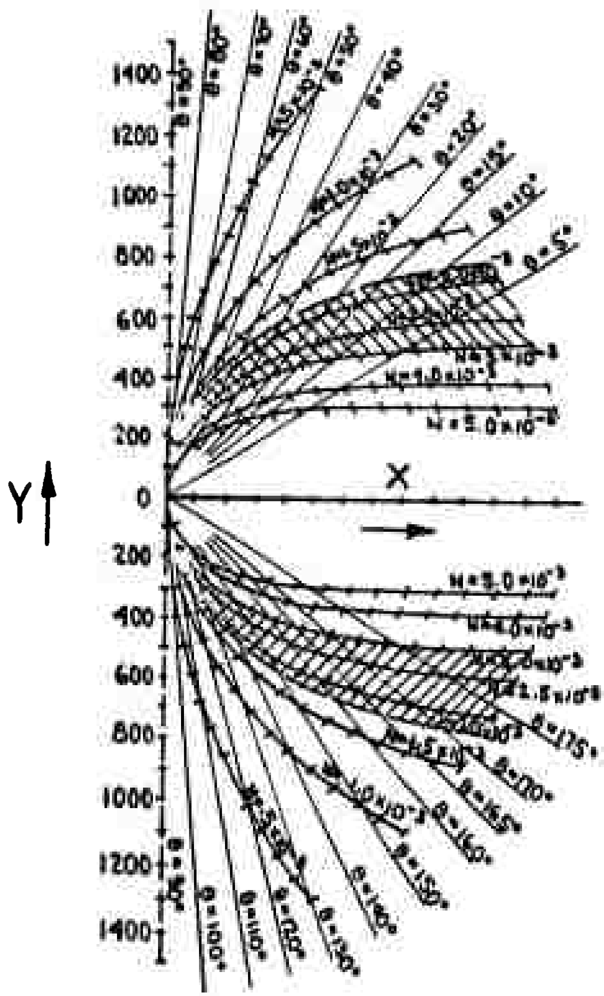
Fig. 11



DEFLECTION PROFILE CHART	GRAPH NO. : 38	
	DRAWN: av	DATE: 10-1-63
	APPR.:	

$$X = -\frac{1}{N} \ln \cos(YN)$$

FiG. 12

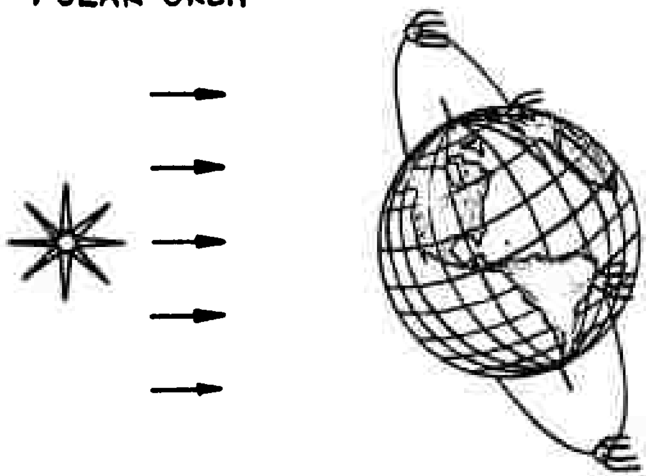


TN 164

DEFLECTION PROFILES OF TUBULAR ELEMENTS IN ORBIT AROUND THE EARTH ELEMENT PROFILE: $N = 1.86 \times 10^{-3}$ TO 3.1×10^{-3}	GRAPH NO.: 44	
	DRAWN: A.W.	DATE: 11-1-63
	APPR.:	

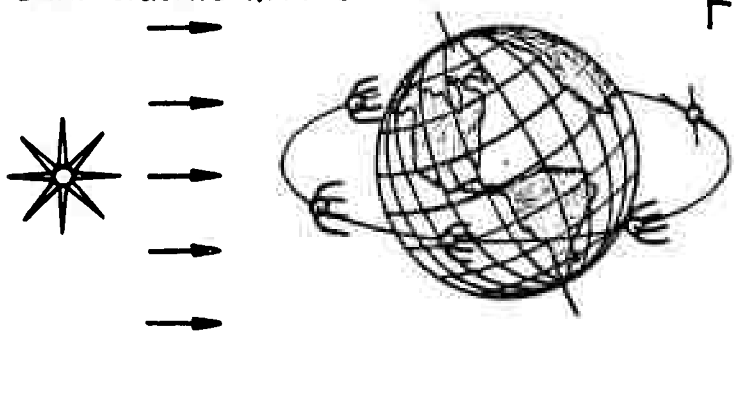
POLAR ORBIT

FIG. 13



SEMI-EQUATORIAL ORBIT

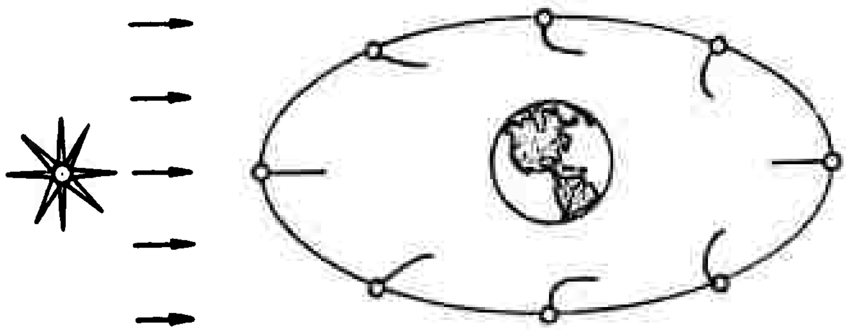
FIG. 14



DEFLECTION PROFILES OF TUBULAR ELEMENTS IN ORBITS AROUND THE EARTH ELEMENT PROFILE: $N = 1.86 \times 10^{-3}$ TO 3.1×10^{-3}	GRAPH NO.: 45	
	DRAWN: AW.	DATE
	APPR.:	11-1-63

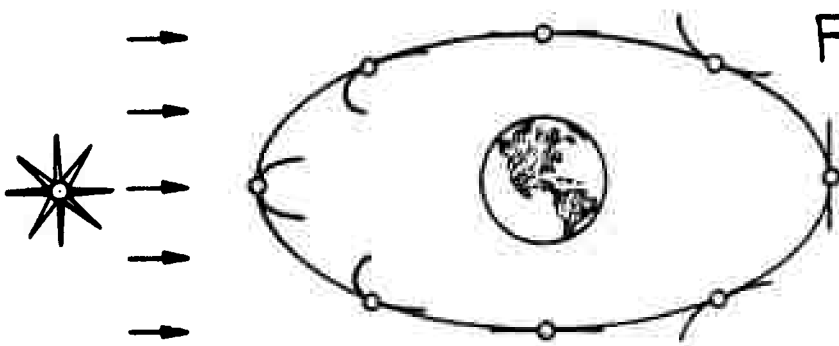
ONE ELEMENT POINTING TO THE EARTH

Fig. 15



DIPOLE ELELEMENTS AS TANGENT TO THE ORBIT

Fig. 16



TN 164

PART II

MATHEMATICAL ANALYSIS

PART II

MATHEMATICAL ANALYSIS

1.0 INTRODUCTION

The purpose of this part of the report is to show the mathematical development of the formulas and equations needed to calculate distorted shapes of long thin-walled tubes subjected to heat radiation in space. For the most part, the equations are developed in general form, but in certain cases where complexities warrant it, numerical values are used to arrive at working equations. In these cases the equations are made to apply to a 0.5 inch diameter beryllium copper tube, 0.002 inches thick, subjected to radiation in space just outside the earth's atmosphere. Thus the principle sources of radiation considered are the sun, which delivers heat at an intensity of 450 BTU/HR-FT^2, and the earth, which delivers its own heat at an intensity of 70 BTU/HR-FT^2 and reflected sun heat at an intensity of 160 BTU/HR-FT^2.

The method of obtaining the required equations is to develope temperature profiles on the tube cross-section by consideration of the various radiation and conduction heat flows. The temperature profiles are then used to determine the thermal distortion of the tube.

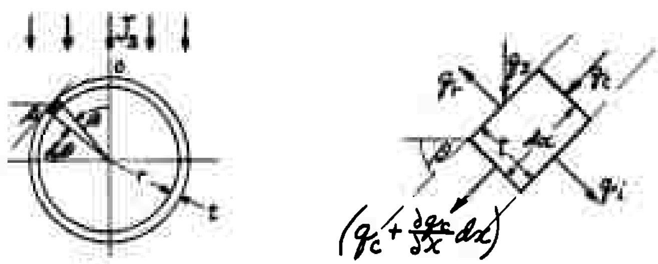
Since the elements are made of flat metal strips formed into circular cross-section tubes with approximately 180 degrees of overlap, it is assumed that there is always some area of contact in the overlapped section. Because of the forming method used in producing the tubes, the most probable place for contact to occur is at the outer free edge of the formed strip. This type of contact makes the overlapped tube similar to a seamless tube from a heat transfer point of view, and for this reason the temperature profile development in section 2.0 deals only with seamless tubes. The effect of increased area of contact in the overlapped section on temperature profiles is discussed in section 5.0.

Equations for the distorted shape of tubular elements are developed in section 3.0. These equations are of interest mainly because they show clearly what factors will increase or decrease element deflections. Actual magnitudes of deflections obtained from these equations are only approxi mate because of the assumptions used in the mathematical development.

2.0 DERIVATION OF TEMPERATURE PROFILES FOR THIN-WALLED CYLINDRICAL TUBES

2.1 Heat Balance

The temperature profile of a thin-walled cylindrical tube can be determined by equating the heat absorbed to the heat rejected for a differential element of the tube. Since the thickness t is much smaller than the radius r, (less than 0.01 r) temperature gradients in the radial direction will be negligible compared to temperature gradients around the perimeter, and the differential element shown in Figure 1 (a) can be changed to a rectangular element of length $dx = rd\beta$, as shown in Figure 1 (b). The heat flows indicated in Figure 1 (b) are defined as follows:



(a) (b)

Figure 1

q_s = Radiant heat from sun or other source absorbed at the surface of the tube.

q_r = Heat radiated to space.

q_c = Heat flow around tube due to conduction.

q_i = Heat radiation across interior of tube.

Heat flow along the length of the tube is assumed to be zero, thus allowing all further calculations to be based on a unit length of tube. This assumption is valid as long as the heat source remains constant over long lengths of the tube.

Equating heat into heat out yields

$$q_s + q_c = q_r + q_i + \left(q_c + \frac{\partial q_c}{\partial x} dx\right)$$

$$-\frac{\partial q_c}{\partial x} dx = q_r + q_i - q_s \qquad (1)$$

The temperature profile can be obtained from equation (1) by substituting for the indicated heat flows.

2.2 Heat Flow From Source Absorbed at Surface

If the source of radiant heat is the sun, the heat flow is:

$$q_s = J_S \alpha_s \, dx \cos\left(\frac{x}{r}\right) \qquad (2)$$

where J_s is the radiation intensity and α_s is the solar absorptivity of the material.

If the source of radiant heat is the earth, the heat flow is:

$$q_s = \left(J_R \alpha_s + J_E \epsilon\right) dx \cos\left(\frac{x}{r}\right) \qquad (3)$$

where J_R is the intensity of sunlight reflected by the earth, J_E is the intensity of infra-red earth radiation and ϵ is the emmissivity of the tube material. It is assumed the absorptivity of any material for earth reflected sunlight is the same as that for direct sunlight.

2.3 Heat Radiated to Space

If a surface radiates to black body (space) surroundings, the heat flow is:

$$q_r = \sigma \epsilon T^4 dx \qquad (4)$$

2.4 Heat Flow Due to Conduction

$$q_c = -kt\frac{dT}{dx}$$

from which:

$$\frac{\partial q_c}{\partial x} = -kt\frac{d^2T}{dx^2} \tag{5}$$

2.5 Heat Radiated Across Interior Of Tube

Consider the interior surface of a tube divided into n elements of length Δx, as shown in Figure 2. The amount of radiation leaving element 1 and arriving at any element p is:

$$q_{1\to p} = J_1\frac{\cos\beta_1 \cos\beta_p(\Delta x_1)(\Delta x_p)}{\pi y^2}$$

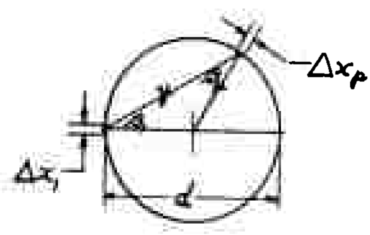


Figure 2

where J_1 is the surface radiosity of element 1. Since the elements lie on a circle, $\beta_1 = \beta_p$, and $y = d\cos\beta_1$. Therefore,

$$q_{1\to p} = J_1\frac{\cos^2\beta_1(\Delta x_1)^2}{\pi d^2\cos^2\beta_1} = J_1\frac{(\Delta x)^2}{\pi d^2}$$

Similarly:

$$q_{p\to 1} = J_p\frac{(\Delta x)^2}{\pi d^2}$$

Thus the net heat flow from 1 to p is:

$$q_{1p} = (J_1 - J_p)\frac{(\Delta x)^2}{\pi d^2}$$

And the total heat flow from 1 to all the other elements is:

$$q_1 = \left[(n-1)J_1 - \sum_2^n J_p\right]\frac{(\Delta x)^2}{\pi d^2} \tag{7}$$

Another expression for the heat leaving element 1 can be derived from the fact that it must be equal to the negative sum of the heat flows leaving the other elements. I.E.:

$$\sum_1^n q = 0$$

$$q_1 = -\sum_2^n q_p \tag{8}$$

Using the general expression for heat leaving the surface of element p

$$q_p = \frac{\epsilon(\Delta x)}{\gamma}\left(E_{b_p} - J_p\right) \tag{9}$$

where E_{b_p} is the black body emissive power of the element and γ is the reflectivity, equation (8) becomes

$$q_1 = -\sum_2^n \left(E_{b_p} - J_p\right)\frac{\epsilon(\Delta x)}{\gamma} \tag{10}$$

Combining equations (7), (9), (10), and realizing that $E_{b_p} = \sigma \epsilon T_p^{\,4}$, gives the following result:

$$q_1 = \left[\frac{(n-1)(\Delta x)\sigma\epsilon T_1^{\,4} - \sigma\epsilon\sum_2^n T_p^{\,4}(\Delta x)}{\pi d^2\epsilon + \gamma n(\Delta x)}\right](\Delta x) \tag{11}$$

If the number of elements is large such that Δx approaches zero, then

$$(n-1)\Delta x \cong n\,dx = \pi d$$

and

$$\sigma\epsilon\sum_{2}^{n} T_p^4(\Delta x) \cong \sigma\epsilon\int_0^{\pi d} T_p^4\,dx = \pi d\,\sigma\epsilon T_m^4$$

where T_m is the mean heat transfer temperature of the tube. Thus the required expression for radiation across the interior of the tube becomes:

$$q_i = \frac{\sigma\epsilon}{\epsilon d + \gamma}\left(T^4 - T_m^4\right)dx \qquad (12)$$

2.6 <u>General Heat Flow Equation</u>

Substitution for the various heat flows in equation (1) gives the general heat balance in terms of temperatures, from which the temperature profile may be obtained. Considering only sun radiation for the moment, equation (1) becomes:

$$kt\frac{d^2T}{dx^2}dx = \sigma\epsilon T^4 dx + \frac{\sigma\epsilon}{\epsilon d+\gamma}\left(T^4 - T_m^4\right)dx - J_s\alpha_s\cos\left(\frac{x}{r}\right)dx$$

from which

$$\frac{d^2T}{dx^2} = \frac{\sigma\epsilon T_m^4}{kt}\left[\left(\frac{T}{T_m}\right)^4 + \left(\frac{1}{\epsilon d+\gamma}\right)\left\{\left(\frac{T}{T_m}\right)^4 - 1\right\}\right] - \frac{J_s\alpha_s}{kt}\cos\left(\frac{x}{r}\right) \qquad (13)$$

Since it is expected that the mean tube temperature will be around 800° R, and that the actual temperature will have a maximum range of $T = T_m \pm 10^\circ$, it can be seen that the function of $(T/T_m)^4$ will range from 1.05 to 1 while cos (x/r) ranges from 1 to 0. Therefore to simplify equation (13) so that an analytic solution may be obtained, it will be assumed that $(T/T_m)^4$ is a constant numerically equal to 1. The heat balance therefore becomes: -

$$\frac{d^2T}{dx^2} = \frac{\sigma\epsilon T_m^4}{kt} - \frac{J_s\alpha_s}{kt}\cos\left(\frac{x}{r}\right) \tag{14}$$

Stated in another way, equation (14) neglects the heat flow across the interior of the tube, and assumes that the heat radiated to space from the tube surface does not vary from point to point around the perimeter.

2.7 Temperature Profile with Sun Radiation Only

If the sun is the only source of radiation impinging on the tube surface, the profile may be obtained from two differential equations. One equation gives the profile in the first quadrant, the other gives the profile in the second quadrant. Figure 3 shows that by symmetry the profiles in the third and fourth quadrants are identical to those in the second and first quadrants respectively.

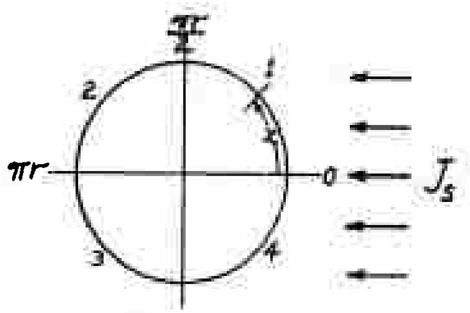


Figure 3

The two equations may be written directly from equation (14) realizing that the source is zero in the second quadrant.

$$\frac{d^2T_1}{dx^2} = \frac{\sigma\epsilon T_m^4}{kt} - \frac{J_s\alpha_s}{kt}\cos\left(\frac{x}{r}\right) \qquad \left[0<x<\frac{\pi r}{2}\right] \tag{15}$$

$$\frac{d^2T}{dx^2} = \frac{\sigma\epsilon T_m^4}{kt} \qquad \left[\frac{\pi r}{2}<x<\pi r\right]$$

Equations (15) can be integrated directly to give:

$$\frac{dT_1}{dx} = \frac{r}{kt}\left[\sigma\epsilon T_m^4\left(\frac{x}{r}\right) - J_s\alpha_s \sin\left(\frac{x}{r}\right) + C_1\right]$$

$$\frac{dT_2}{dx} = \frac{r}{kt}\left[\sigma\epsilon T_m^4\left(\frac{x}{r}\right) + C_2\right] \qquad (16)$$

$$T_1 = \frac{r^2}{kt}\left[\frac{\sigma\epsilon T_m^4}{2}\left(\frac{x}{r}\right)^2 + J_s\alpha_s \cos\left(\frac{x}{r}\right) + C_1\left(\frac{x}{r}\right) + C_3\right]$$

$$T_2 = \frac{r^2}{kt}\left[\frac{\sigma\epsilon T_m^4}{2}\left(\frac{x}{r}\right)^2 + C_2\left(\frac{x}{r}\right) + C_4\right]$$

The constants of integration in equations (16) can be solved for by applying the boundary conditions of continuous heat flow and temperature at $x = \pi r/2$, and zero heat flow at $x = 0$ and $x = \pi r$. Stated mathematically:

$$\frac{dT_1}{dx}(x=0) = \frac{dT_2}{dx}(x=\pi r) = 0$$

$$\frac{dT_1}{dx}\left(x=\frac{\pi r}{2}\right) = \frac{dT_2}{dx}\left(x=\frac{\pi r}{2}\right)$$

$$T_1\left(x=\frac{\pi r}{2}\right) = T_2\left(x=\frac{\pi r}{2}\right)$$

from which:

$$T_1 = \frac{r^2}{kt}\left[\frac{\sigma\epsilon T_m^4}{2}\left(\frac{x}{r}\right)^2 + J_s\alpha_s \cos\left(\frac{x}{r}\right) + C\right] \qquad (17)$$

$$T_2 = \frac{r^2}{kt}\left[\frac{\sigma\epsilon T_m^4}{2}\left(\pi - \frac{x}{r}\right)^2 + C\right]$$

The unknown constant C can be seen by inspection to be the lowest temperature on the cross section. It can be evaluated in terms of the mean heat transfer temperature by means of the following expression:

$$\pi r T_m^4 = \int_0^{\frac{\pi r}{2}} T_1^4\,dx + \int_{\frac{\pi r}{2}}^{\pi r} T_2^4\,dx$$

Because the variation of T_1 and T_2 is small compared to the value of T_m, this can be approximated by:

$$T_m = \frac{1}{\pi r}\left[\int_0^{\frac{\pi r}{2}} T_1\,dx + \int_{\frac{\pi r}{2}}^{\pi r} T_2\,dx\right] \tag{18}$$

Using equation (18) to evaluate C, and substituting C back into equations (17) gives the desired temperature profile. A further substitution can be made from an overall heat balance of heat absorbed to heat radiated:

$$J_s \alpha_s = \pi r \epsilon T_m^4$$

The equations for temperature therefore become:

$$\begin{aligned} T_1 &= T_m + \frac{r^2}{kt}\left[\frac{1}{2\pi}\left(\frac{x}{r}\right)^2 + \cos\left(\frac{x}{r}\right) - 0.45\right] \\ T_2 &= T_m + \frac{r^2}{kt}\left[\frac{1}{2\pi}\left(\pi - \frac{x}{r}\right)^2 - 0.45\right] \end{aligned} \tag{19}$$

Inspection shows that the maximum temperature always occurs at x = 0, and the minimum at x = π r. The equation for maximum temperature difference is therefore:

$$T_{1max} - T_{2min} = \Delta T_{max} = \frac{r^2}{kt} J_s \alpha_s \tag{20}$$

2.8 Temperature Profile with Sun and Earth Radiation Directly Opposed

If sun and earth radiation are directly opposed, the problem may be solved by two differential equations, since the profile will still be symmetrical as shown by Figure 4.

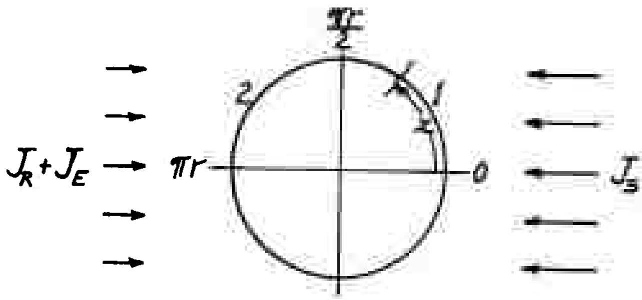


Figure 4

The two equations may be written directly from equation (14) as follows:

$$\frac{d^2T_1}{dx^2} = \frac{\sigma\epsilon T_m^4}{kt} - \frac{J_S\alpha_S}{kt}\cos\left(\frac{x}{r}\right) \qquad \left[0 < x < \frac{\pi r}{2}\right] \tag{15}$$

$$\frac{d^2T_2}{dx^2} = \frac{\sigma\epsilon T_m^4}{kt} + \frac{1}{kt}\left(J_R\alpha_S + J_E\epsilon\right)\cos\left(\frac{x}{r}\right) \qquad \left[\frac{\pi r}{2} < x < \pi r\right]$$

Replacing $J_S \alpha_S$ by S_1, and $(J_R \alpha_S + J_E \epsilon)$ by S_2,

the temperature profile may be obtained by direct integration of equations (15), the boundary conditions being exactly the same as with sun radiation only. The desired profile is therefore:

$$T_1 = T_m + \frac{r^2}{kt}(S_1 + S_2)\left[\frac{1}{2\pi}\left(\frac{x}{r}\right)^2 + \frac{S_1}{S_1+S_2}\cos\left(\frac{x}{r}\right) - 0.45\right] \tag{16}$$

$$T_2 = T_m + \frac{r^2}{kt}(S_1 + S_2)\left[\frac{1}{2\pi}\left(\pi - \frac{x}{r}\right)^2 + \frac{S_2}{S_1+S_2}\cos\left(\frac{x}{r}\right) - 0.45\right]$$

It is not obvious from equations (16) where the maximum and minimum temperatures occur in the profile, the positions depending to a large extend on the relative values of S_1 and S_2. To determine the maxima and minima, equations (16) must be differentiated and set equal to zero:

$$\frac{dT_1}{dx} = 0 = \frac{1}{\pi r}\left(\frac{x}{r}\right) - \frac{1}{r}\left(\frac{S_1}{S_1 + S_2}\right)\sin\left(\frac{x}{r}\right) \qquad \text{a)}$$

$$\frac{dT_2}{dx} = 0 = -\frac{1}{\pi r}\left(\pi - \frac{x}{r}\right) + \frac{1}{r}\left(\frac{S_2}{S_1 + S_2}\right)\sin\left(\frac{x}{r}\right) \qquad \text{b)}$$

from which:

$$\frac{S_1}{S_2} = \frac{1}{\left[\pi \sin\left(\frac{x}{r}\right)/\left(\frac{x}{r}\right)\right] - 1} = \frac{J_S \alpha_S}{J_R \alpha_S + J_E \epsilon} \qquad \text{a)}$$

$$\frac{S_1}{S_2} = \left[\pi \sin\left(\frac{x}{r}\right)/\left(\frac{x}{r}\right)\right] - 1 = \frac{J_S \alpha_S}{J_R \alpha_S + J_E \epsilon} \qquad \text{b)}$$

Using the values of J_S, J_R and J_E which occur at the outer edges of the earth's atmosphere, values of α_S / ϵ can be obtained from equations a) and b) for various values of (x/r).

$$\left(\frac{x}{r}\right) = 0\ ; \qquad \frac{S_1}{S_2} = \frac{1}{\pi - 1} \qquad \frac{\alpha_S}{\epsilon} = 0.087 \qquad \text{a)}$$

$$\left(\frac{x}{r}\right) = \frac{\pi}{2}\ ; \qquad \frac{S_1}{S_2} = 1 \qquad \frac{\alpha_S}{\epsilon} = 0.242 \qquad \text{a) b)}$$

$$\left(\frac{x}{r}\right) = \pi\ ; \qquad \frac{S_1}{S_2} = \pi - 1 \qquad \frac{\alpha_S}{\epsilon} = 1.39 \qquad \text{b)}$$

The significance of these results is that they give the range of values of α_S/ϵ for which the minimum temperature lies in a certain quadrant. From the boundary conditions of the problem, it is known that the maximum temperature must be at either $x = 0$ or $x = \pi r$. The result of equation b) at $x = \pi r$ shows that the minimum temperature will be at $x = \pi r$ for values of $\alpha_S / \epsilon > 1.39$, which is the range that covers most polished metal surfaces. It can be seen, therefore, that the maximum temperature difference across a tube for directly opposed sun and earth radiation will be: -

$$\Delta T_{max} = T_1(x=0) - T_2(x=\pi r)$$

$$\Delta T_{max} = \frac{r^2}{kt}\left[J_s \alpha_s - \left(J_R \alpha_s + J_E \epsilon\right)\right] \qquad (17)$$

2.9 Temperature Profile With Sun and Earth Radiation Not Directly Opposed

If the tube is oriented such that the sun and earth radiation directions have an angle δ between, the problem of deriving the temperature profile becomes more complicated. Four differential equations are required, corresponding to the four segments shown in Figure 5.

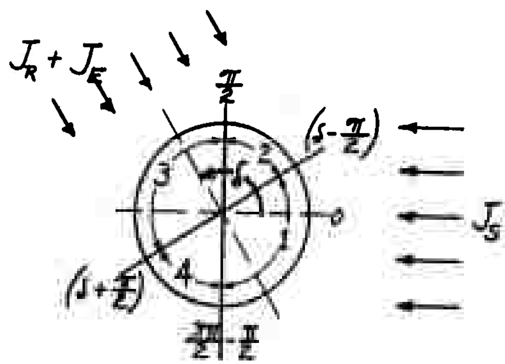


Figure 5

The four equations may be written directly from equation (14) as follows:

$$\frac{d^2T_1}{dx^2} = \frac{\sigma \epsilon T_m^4}{kt} - \frac{S_1}{kt}\cos\left(\frac{x}{r}\right) \qquad \left[-\frac{\pi}{2} < \frac{x}{r} < \delta - \frac{\pi}{2}\right] \qquad (18)$$

$$\frac{d^2T_2}{dx^2} = \frac{\sigma \epsilon T_m^4}{kt} - \frac{S_1}{kt}\cos\left(\frac{x}{r}\right) - \frac{S_2}{kt}\cos\left(\delta - \frac{x}{r}\right) \qquad \left[\delta - \frac{\pi}{2} < \frac{x}{r} < \frac{\pi}{2}\right]$$

$$\frac{d^2T_3}{dx^2} = \frac{\sigma\epsilon T_m^{\,4}}{kt} - \frac{S_2}{kt}\cos\left(\delta - \frac{x}{r}\right) \qquad \left[\frac{\pi}{2} < \frac{x}{r} < \delta + \frac{\pi}{2}\right]$$

$$\frac{d^2T_4}{dx^2} = \frac{\sigma\epsilon T_m^{\,4}}{kt} \qquad \left[\delta + \frac{\pi}{2} < \frac{x}{r} < \frac{3\pi}{2}\right]$$

Equations (18) may be integrated directly to give the temperature profile, the boundary conditions being continuity of temperature and heat flow between the segments.

$$T_1 = \frac{r^2}{kt}\left[\frac{S_1+S_2}{2\pi}\left(\frac{x}{r}\right)^2 + S_1\cos\left(\frac{x}{r}\right) + S_2(1-\delta)\left(\frac{x}{r}\right) - 2S_2\delta - S_1\frac{\pi}{2} + C\right] \tag{19}$$

$$T_2 = \frac{r^2}{kt}\left[\frac{S_1+S_2}{2\pi}\left(\frac{x}{r}\right)^2 + S_1\cos\left(\frac{x}{r}\right) + S_2\cos\left(\delta-\frac{x}{r}\right) - S_2\left(\frac{\delta}{\pi}\right)\left(\frac{x}{r}\right) - S_2\left(\delta+\frac{\pi}{2}\right) - S_1\left(\frac{\pi}{2}\right) + C\right]$$

$$T_3 = \frac{r^2}{kt}\left[\frac{S_1+S_2}{2\pi}\left(\frac{x}{r}\right)^2 + S_2\cos\left(\delta-\frac{x}{r}\right) + \left(S_1+S_2\frac{\delta}{\pi}\right)\left(\frac{x}{r}\right) - S_2\left(\delta+\frac{\pi}{2}\right) + C\right]$$

$$T_4 = \frac{r^2}{kt}\left[\frac{S_1+S_2}{2\pi}\left(\frac{x}{r}\right)^2 - S_1\left(\frac{x}{r}\right) - S_2\left(1+\frac{\delta}{\pi}\right)\left(\frac{x}{r}\right) + C\right]$$

To obtain the maximum temperature difference across the cross section of the tube, the positions of maximum and minimum temperature must first be established by differentiating equations (19) and setting the derivatives equal to zero.

$$\frac{dT_1}{dx} = 0 = \frac{S_1+S_2}{\pi}\left(\frac{x}{r}\right) - S_1\sin\left(\frac{x}{r}\right) + S_2\left(1-\frac{\delta}{\pi}\right) \tag{20}$$

$$\frac{dT_2}{dx} = 0 = \frac{S_1+S_2}{\pi}\left(\frac{x}{r}\right) - S_1\sin\left(\frac{x}{r}\right) + S_2\sin\left(\delta-\frac{x}{r}\right) - S_2\left(\frac{\delta}{\pi}\right)$$

$$\frac{dT_3}{dx} = 0 = \frac{S_1+S_2}{\pi}\left(\frac{x}{r}\right) + S_2 \sin\left(\delta - \frac{x}{r}\right) - \left(S_1 + S_2 \frac{\delta}{\pi}\right)$$

$$\frac{dT_4}{dx} = 0 = \frac{S_1+S_2}{\pi}\left(\frac{x}{r}\right) - S_1 - S_2\left(1 + \frac{\delta}{\pi}\right)$$

It can be seen from equations (20) that the values of x/r to give maxima or minima depend on the angle δ and the ratios $S_1/S_1 + S_2$ and $S_2/S_1 + S_2$. These two ratios may be considered constant if the ratio α_s/ϵ is greater than 3, which is the case that will be considered here. It can also be seen from Figure 5 and the results of Sections 2.8 that the maximum temperature must occur in segment 1 or 2, and the minimum in segment 3 or 4. Figure 6 shows the functions $(x/r)_{max.}$ and $(x/r)_{min.}$ plotted against δ, and also shows the segments in which they occur.

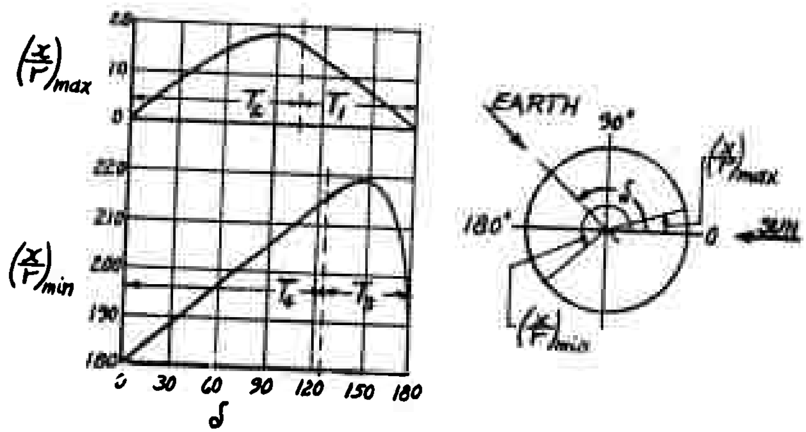


Figure 6

By substituting values of $(x/r)_{max.}$ and $(x/r)_{min.}$ back into equations (20), the maximum temperature difference across the tube can be obtained for any value of δ. In general, an expression for this temperature difference would be of the form:

$$\Delta T_{max} = \frac{r^2}{kt} f(S_1, S_2, \delta) \tag{21}$$

where f indicates an unknown functional relationship. Taking S_1 and S_2 to be constant in this analysis, $\Delta T_{max.}$ can be plotted as a function of δ. Figure 7 shows that the functional relationship approaches a cosine squared curve, such that temperature difference across the tube can be approximated by the formula:

$$\Delta T_{max} = \frac{r^2}{kt}\left(S_1 + S_2 \cos^2\delta\right) \qquad \left[0<\delta<\frac{\pi}{2}\right] \quad (22)$$

$$\Delta T_{max} = \frac{r^2}{kt}\left(S_1 - S_2 \cos^2\delta\right) \qquad \left[\frac{\pi}{2}<\delta<\pi\right]$$

For an element on a satellite orbiting the earth, the assumption that S_2 is a constant is not correct, since it will change with distance from the earth's surface and with position relative to the earth's bright side. However, numerical computations for differing values of S_2 show that equation (22) is reasonably accurate regardless of the value of S_2, in the region $\delta > 90^\circ$.

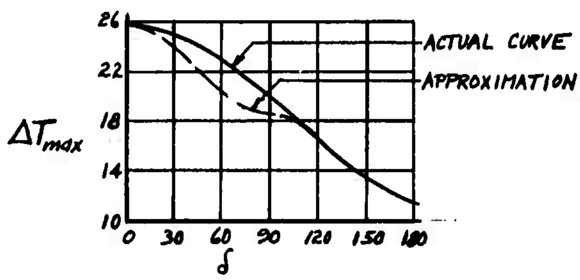


Figure 7

3.0 DERIVATION OF THERMAL DISTORTION

3.1 Thermal Distortion Due to Sun and Directly Opposed Sun and Earth Radiation

It has been shown that a thin-walled tube in space will develope a temperature difference across its cross-section due to radiation incident on its surface. Because of this temperature difference, longitudinal strips

of the tube will expand to different lengths, causing distortion of the tube. Consider a short length of such a tube, as shown in Figure 8. If the temperature gradient across the tube is linear with respect to a diameter passing through the points of maximum and minimum temperature, the distortion will be uniform bending about a radius of curvature R_c, and there will be no distortion of the cross-section.

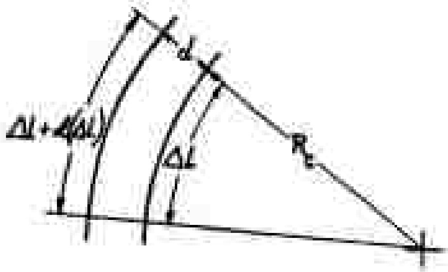


Figure 8

From the geometry of Figure 8

$$\frac{\Delta l}{R_c} = \frac{\Delta l + d(\Delta l)}{R_c + d}$$

From which

$$\frac{1}{R_c} = \frac{1}{d} \times \frac{d(\Delta l)}{\Delta l} = \frac{1}{d} e \Delta T \qquad (23)$$

where e is the coefficient of thermal expansion of the tube material and ΔT is the temperature difference across the diameter.

Consider now a long tube fixed at one end with radiation striking it at an angle θ_e to the fixed end, as shown in Figure 9.

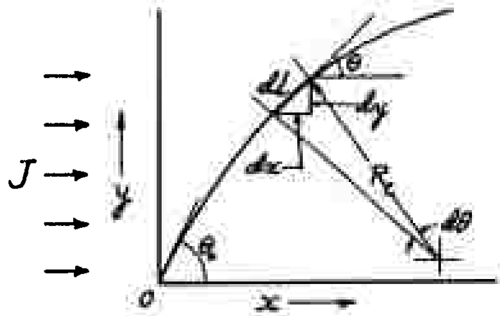


Figure 9

If heat flow along the length of the tube is neglected, the temperature difference across the tube at any angle θ is:

$$\Delta T = \Delta T_{max} \sin\theta \tag{24}$$

where $\Delta T_{max.}$ is the temperature difference at the fixed end.

Considering an element of length dl at any angle θ :

$$\begin{aligned} -R_c \, d\theta &= dl \\ \frac{1}{R_c} = -\frac{d\theta}{dl} &= -\frac{d\theta}{dy}\sin\theta \end{aligned} \tag{25}$$

Substituting equation (24) into (23) and equating to (25) gives the following:

$$\begin{aligned} -\frac{d\theta}{dy}\sin\theta &= \frac{e\Delta T_{max}}{d}\sin\theta \\ d\theta &= -N\,dy \end{aligned} \tag{26}$$

where $N = e \Delta T_{max}/d$. Integrating equation (26) and applying the condition that at $y = o$, $\theta = \theta_o$;

$$\theta = \theta_o - Ny \tag{27}$$

Taking the contangent of equation (27) gives:

$$\cot\theta = \frac{dx}{dy} = \cot(\theta_o - Ny)$$

$$dx = \cot(\theta_o - Ny)\,dy \tag{28}$$

Integrating equation (28) and applying the condition that at $y = o$, $x = o$;

$$x = -\frac{1}{N} Ln\left[\frac{\sin(\theta_o - Ny)}{\sin\theta_o}\right] \tag{29}$$

Equation (29) is the equation for the shape a tube will take wh... radiation causes a linear temperature gradient along a diameter. Since θ_o can have any value from 0 to 180 degrees, it is useful to take $\theta_o = \pi/2$ in which case equation (29) becomes

$$x = -\frac{1}{N} Ln \cos(Ny) \tag{30}$$

Equation (30) can then be plotted as shown in Figure 5, Part I of this report, with the y axis positive in either direction from zero, and the angles θ indicated at the appropriate points on the curve. In this way, the factor N is calculated from the value of $\Delta T_{max.}$ obtained by considering the radiation to be at right angles to the tube.

Distance along the curve in unit lengths of tube is also plotted in Figure 5, Part I. This is obtained by combining equations (24) and (25) in a slightly different manner to give

$$\frac{1}{R_c} = -\frac{d\theta}{dl} = N \sin\theta$$

$$dl = -\frac{1}{N} \csc\theta \, d\theta \tag{31}$$

Integrating equation (31) and applying the condition (from Figure 5, Part I) that at $l = 0$, $\theta = \pi/2$

$$l = -\frac{1}{N} \ln \tan\frac{\theta}{2} \tag{32}$$

For short tubes, Figure 5, Part I, shows that deflections are small compared to the tube length. Thus the ordinate y in equation (30) may be replaced by distance l along the tube, giving the deflection x as

$$x = -\frac{1}{N} \ln \cos(Nl) \tag{33}$$

In particular if $Nl \leqslant 0.3$, equation (33) simplifies to

$$x = \frac{N}{2} l^2 \tag{34}$$

with a deviation of less than 2%.

Equation (34) or Figure 5, Part I, may be used to give the distortion shape of a tube with a temperature difference across its cross-section induced either by sun radiation or by directly opposed earth and sun radiation. However, it must be remembered that the shape is only approximate,

since actual temperature gradients with respect to a diameter are not linear as was first stipulated.

Figure 11 shows some typical temperature profiles plotted against tube diameter. It is expected that the small deviations from a linear gradient will cause distortion of the cross-section, thus changing somewhat the bending shape of the tube. To calculate the final bending shape would require the

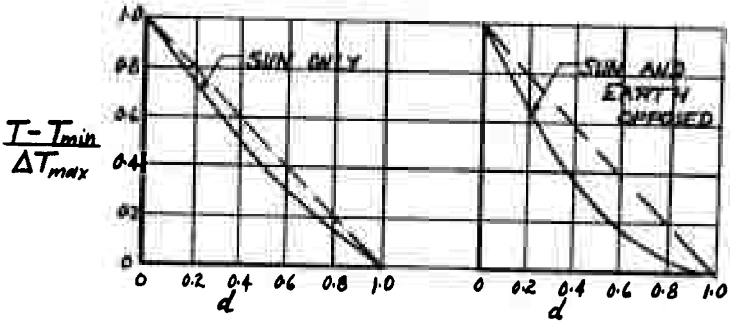


Figure 11

minimizing of lengthwise and cross-sectional strain energies to determine the shape of least energy. However, this final shape would not be markedly different from that already derived. Therefore the task of deriving it will not be undertaken in this report.

3.2 Thermal Distortion Due to Sun and Earth Radiation Not Directly Opposed

If the sun and earth radiations are not directly opposed, the problem of determining the distortion shape becomes more complicated. Not only is the temperature gradient non-linear along a diameter, the points of maximum and minimum temperature do not lie on a diameter. However, the problem can be simplified somewhat by assuming that the temperature gradient due to a single source is linear along a diameter. Since temperature profiles are solutions of linear differential equations, the profile due to two sources with an angle δ between is the sum of two profiles due to the sources acting separately. Also, since the separate profiles are assumed linear, the temperature difference across the tube in the direction of the sun radiation is proportional to $S_1 + S_2 \cos \delta$ (see figure 12) and the temperature difference at right angles to the direction of sun radiation is proportional to $S_2 \sin \delta$.

In this manner any problem involving two radiation sources can be transformed into a problem of two different sources acting at right angles to each other.

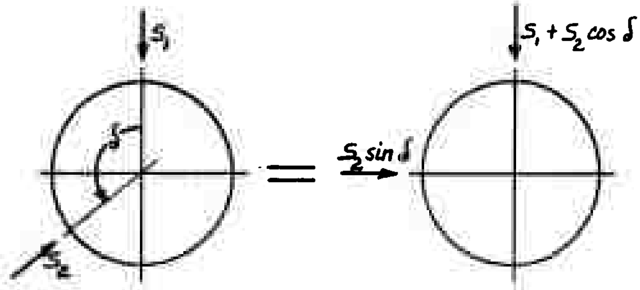


Figure 12

Consider now a differential length dl of the tube in space, as shown in Figure 12. The effective source S_3 will cause bending of the tube about a

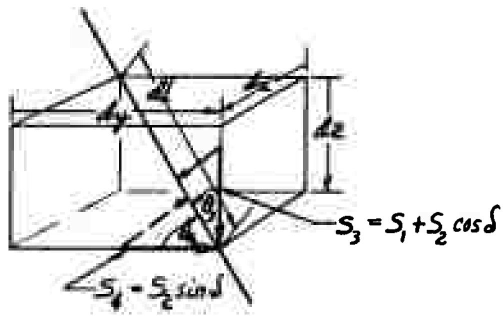


Figure 13

radius of curvature R_{c_3} in the direction of $S_3 \sin \theta_3$. Similarly S_4 will cause bending about a radius R_{c4} in the direction of $S_4 \sin \theta_4$. Using equations previously developed:

$$\frac{1}{R_{c_3}} = -\frac{d\theta_3}{dl} = -\frac{d\theta_3}{dz}\cos\theta_3 = \frac{e\Delta T_{3max}}{d}\sin\theta_3 \qquad (35)$$

$$\frac{1}{R_{c4}} = -\frac{d\theta_4}{dl} = -\frac{d\theta_4}{dy}\cos\theta_4 = \frac{e\Delta T_{4max}}{d}\sin\theta_4$$

where

$$\Delta T_{3max} = \frac{r^2}{kt}\left(S_1 + S_2 \cos \delta\right)$$

$$\Delta T_{4max} = \frac{r^2}{kt}\left(S_2 \sin \delta\right)$$

Integrating equations (35) and applying the conditions that at y = z = o, $\theta_3 = \theta_4 = \pi/2$;

$$z = -\frac{1}{N_3} \ln \sin \theta_3$$

$$y = -\frac{1}{N_4} \ln \sin \theta_4 \qquad (36)$$

where

$$N_3 = \frac{e\,\Delta T_{3max}}{d}$$

$$N_4 = \frac{e\,\Delta T_{4max}}{d}$$

Equations (35) may be integrated in terms of the length l, and then solved for in terms of θ_3 and θ_4 to give:

$$\theta_3 = 2 \arctan\left[e^{-N_3 l}\right] \qquad (37)$$

$$\theta_4 = 2 \arctan\left[e^{-N_4 l}\right]$$

Equations (36) and (37) may be used to solve for the components of deflection (y and z) at any distance l along a tube which is being deformed by two sources of radiation with an angle δ between, each initially at right angles to the tube. The shape of the deformed tube is shown in Figure 14.

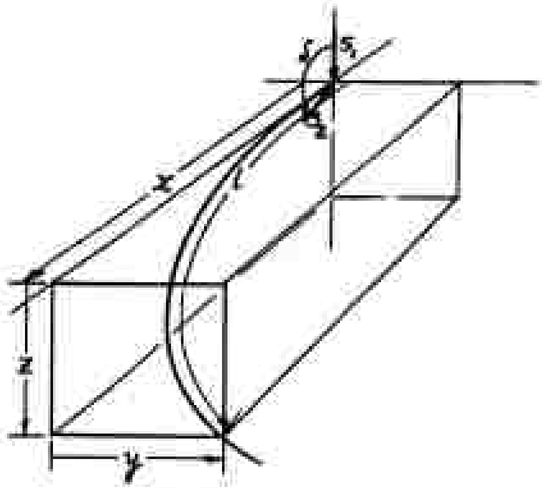


Figure 14

Again it must be pointed out that the equations for y and z give approximate deflections because of the assumptions stipulated in the derivation.

4.0 DERIVATION OF ELEMENT RESPONSE TIME

An estimate of the length of time required for an element to build up a temperature gradient across its cross-section can be obtained by investigating the response of the mean tube temperature to sudden exposure of the tube to sun radiation. This assumes that the temperature gradients calculated in Section 2.0 will be realized when the element is in thermal equilibrium.

Consider an element initially at a uniform temperature T_{m_o}. If it is suddenly placed in sunlight, a balance of the heat flows can be made as follows:

Heat Radiation In — Heat radiated out = Heat absorbed.

Substituting for the heat flows gives the following:

$$dJ_s\alpha_s - \pi d\sigma\epsilon T_m^4 = c\rho\phi\pi d t \frac{dT}{d\tau} \tag{38}$$

where c is the specific heat of the material, ϕ is the overlap factor and τ is time.

Equation (38) can be re-arranged to allow numerical integration. Substituting for the sun radiation in terms of the final equilibrium mean temperature T_{m_f} gives the required expression:

$$\Delta T_m = \frac{\sigma\epsilon}{c\rho\phi t}\left(T_{m_f}^4 - T_m^4\right)\Delta\tau \tag{39}$$

The temperature T_m plotted as a function of time τ is shown in Figure 10, Part I, of this report, for a temperature increase $(T_{m_f} - T_{m_o})$ of 285°F.

If the step increase in temperature is small such that $T_m = T_{m_o} + \Delta T_m$, where ΔT_m is very small compared to T_{m_o}, then equation (39) can be simplified to:

$$\frac{dT_m}{d\tau} = \frac{d(\Delta T_m)}{d\tau} = \frac{\sigma\epsilon}{c\rho\phi t}\left[\left(T_{m_o} + \Delta T_{max}\right)^4 - \left(T_{m_o} + \Delta T_m\right)^4\right] \tag{40}$$

Since ΔT_m is small,

$$\left(T_{m_o} + \Delta T_{max}\right)^4 - \left(T_{m_o} + \Delta T_m\right)^4 = 4T_{m_o}^3\left(\Delta T_{max} - \Delta T_m\right)$$

and equation (40) reduces to:

$$\frac{d(\Delta T_m)}{d\tau} + \frac{4\sigma\epsilon T_{m_o}^3}{c\rho\phi t}\left(\Delta T_m\right) = \frac{4\sigma\epsilon T_{m_o}^3}{c\rho\phi t}\left(\Delta T_{max}\right) \tag{41}$$

The solution to equation (41) is:

$$\Delta T_m = \Delta T_{max}\left[1 - e^{-(\tau/C_\tau)}\right]$$

where

$$C_\tau = \frac{c\rho\delta t}{4\sigma\epsilon T_m^3} \qquad (42)$$

and is referred to as the time constant of the element.

The value of the time constant C_τ is of interest since it represents the time required for the element to reach 63.2% of its maximum temperature rise, and can be used in a stability analysis of an antenna which undergoes a small change in orientation with respect to the radiation heat source.

5.0 DISCUSSION OF ASSUMPTIONS

5.1 Interior Radiation and Surface Radiation to Space

Two assumptions made in the derivations of the temperature profiles were that the interior radiation could be neglected and that the tube would radiate to space at its mean temperature. To determine the validity of these assumptions, the heat flow equation will be solved again for the simple case of sun radiation only, taking all heat flows into account. Equation (13) may be rewritten in the following way:

$$\frac{d^2T}{dx^2} - \frac{\sigma\epsilon}{kt}\left(1 + \frac{1}{\epsilon d + \gamma}\right)\left(T^4 - T_m^4\right) = \frac{\sigma\epsilon T_m^4}{kt} - \frac{J_s\alpha_s}{kt}\cos\left(\frac{x}{r}\right) \qquad (43)$$

Since the difference between T and T_m is very small, the following simplification may be used, with an error of less than 2%.

$$T^4 - T_m^{\,4} = \left[T_m + (T - T_m)^4\right] - T_m^{\,4} = 4T_m^{\,3}(T - T_m)$$

Therefore equation (43) becomes:

$$\frac{d^2T}{dx^2} - \left[\frac{\sigma\epsilon T_m^{\,3}}{kt}\left(4 + \frac{4}{\epsilon d + \gamma}\right)\right]T = -\left[\frac{\sigma\epsilon T_m^{\,3}}{kt}\left(3 + \frac{4}{\epsilon d + \gamma}\right)\right] - \frac{J_s \alpha_s}{kt}\cos\left(\frac{x}{r}\right) \qquad (44)$$

Replacing the constants in equation (44) by a^2, b and d, the two equations of heat flow for sun radiation only acting on the tube may be written as follows:

$$\frac{d^2T_1}{dx^2} - a^2T_1 = -b - d\cos\left(\frac{x}{r}\right) \qquad \left[0 < x < \frac{\pi r}{2}\right]$$

$$\frac{d^2T_2}{dx^2} - a^2T_2 = -b \qquad \left[\frac{\pi r}{2} < x < \pi r\right] \qquad (45)$$

The solution to equations (45) is:

$$T_1 = \frac{rd}{2a(1+a^2r^2)}\frac{\cosh(ax)}{\sinh\left(\frac{a\pi r}{2}\right)} + \frac{b}{a^2} + \frac{r^2d}{1+a^2r^2}\cos\left(\frac{x}{r}\right)$$

$$T_2 = \frac{rd}{2a(1+a^2r^2)}\left[\frac{\cosh(ax)}{\sinh\left(\frac{a\pi r}{2}\right)} - 2\sinh a\left(x - \frac{\pi r}{2}\right)\right] + \frac{b}{a^2} \qquad (46)$$

The maximum temperature difference across the tube is therefore:

$$T_1(x=0) - T_2(x=\pi r) = \Delta T_{max} = \frac{r^2d}{1+a^2r^2}$$

$$\Delta T_{max} = f\,\frac{r^2}{kt}J_s\alpha_s \qquad (47)$$

where

$$f = \frac{1}{1+a^2r^2} = \frac{1}{1+\left(4+\frac{4}{\epsilon\alpha+\gamma}\right)\frac{r^2}{kt}\sigma\epsilon T_m^3}$$

It can be seen that the factor f is always slightly smaller than unity. For the particular case of a one-half inch beryllium copper tube, 0.002 inches thick, f has a value of 0.96.

It is therefore to be expected that neglecting interior radiation in the tube and assuming radiation to space at the mean temperature of the tube will give temperature differences across the tube which are approximately 4% too high.

5.2 Heat Flow Along the Tube Length

Another assumption made in the derivation of the temperature profile was that the heat flow along the length of the tube is zero. However, it can be seen that because of the distorted tube shape, the mean temperature varies along the tube length according to the formula:

$$T_m^4 = T_{m_0}^4 \sin\theta \qquad (48)$$

where T_{m_0} is the mean temperature at the fixed end, and θ is the angle at any point on the tube, as shown in Figure 9.

To obtain the average heat flux, equation (48) must be differentiated with respect to length l and multiplied by the conductivity k :

$$-k\frac{dT_m}{dl} = -\frac{k}{4}T_{m_0}(\sin\theta)^{-\frac{3}{4}}\cos\theta\frac{d\theta}{dl} \qquad (49)$$

Substituting for $d\theta/dl$ from equation (31) gives:

$$-k\frac{dT_m}{dl} = \frac{kNT_{mo}}{4}(\sin\theta)^{\frac{1}{4}}\cos\theta \qquad (50)$$

Equation (50) can be differentiated with respect to θ and set equal to zero to find the point of maximum heat flux along the length. If this is done, it is found that θ_{max} equals 26.6 degrees, and the expression for maximum heat flux becomes:

$$\left(-k\frac{dT_m}{dl}\right)_{max} = 0.183\, kNT_{m_o} \qquad (51)$$

The average heat flux around the perimeter of the tube at θ equals 26.6 degrees is:

$$-k\frac{dT}{dx} = \frac{k\Delta T_{max}\sin(26.6^\circ)}{0.5\,\pi d} \qquad (52)$$

Taking a ratio of equation (51) to (52), and substituting for N, gives, for the beryllium copper tube;

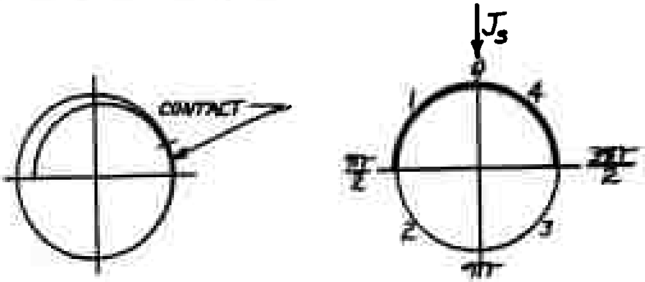
$$\frac{\text{flux along tube length}}{\text{flux around tube perimeter}} = 0.64\, e\, T_{m_o} \cong .005$$

This result indicates that the heat flux along the length of the tube is less than one-half percent of the average heat flux around the perimeter of the tube, and will therefore have a negligible effect on the temperature distributions calculated in section 2.

5.3 Overlapped Tubes

In deriving temperature profiles in Section 2, it was assumed that the tube was seamless and of constant thickness. If the tube is overlapped 180 degrees, this assumption implies that there is enough contact in the overlapped section to give a conduction heat flow area equivalent to the constant area of the seamless tube. The amount of contact required in this

case would be small, as illustrated in Figure 15 (a). If the contact area were larger, as shown in Figure 15 (b), the problem would be similar to a seamless tube having one side twice the thickness of the other. For this case, the temperature difference



(a) (b)

Figure 15

across the tube will be derived with sun radiation at x = 0, $\pi r/2$ and πr.

When the sun radiation is at x = 0, the problem is symmetrical and the two equations of heat flow can be written as follows:

$$\frac{d^2T_1}{dx^2} = \frac{\sigma\epsilon T_m^4}{2kt} - \frac{J_s\alpha_s}{2kt}\cos\left(\frac{x}{r}\right) \qquad \left[0 < x < \frac{\pi r}{2}\right] \qquad (53)$$

$$\frac{d^2T_2}{dx^2} = \frac{\sigma\epsilon T_m^4}{kt} \qquad \left[\frac{\pi r}{2} < x < \pi r\right]$$

Equations (53) can be integrated as in Section 2, with the same boundary conditions, to give:

$$T_1 = \frac{r^2}{kt}J_s\alpha_s\left[\frac{1}{4\pi}\left(\frac{x}{r}\right)^2 + \frac{1}{2}\cos\left(\frac{x}{r}\right) + \frac{1}{4\pi}\left(\frac{\pi}{2}\right)^2 + C\right] \qquad (54)$$

$$T_2 = \frac{r^2}{kt}J_s\alpha_s\left[\frac{1}{2\pi}\left(\pi - \frac{x}{r}\right)^2 + C\right]$$

The maximum temperature difference across the tube is therefore:

$$T_1(x=0) - T_2(x=\pi r) = \Delta T_{max} = \frac{r^2}{kt} J_s \alpha_s \left(\frac{1}{2} + \frac{\pi}{16}\right)$$

$$\Delta T_{max} = 0.696 \frac{r^2}{kt} J_s \alpha_s \qquad (55)$$

Similarly it can be shown that when the sun radiation is at x = $\pi r/2$, the maximum temperature difference is:

$$\Delta T_{max} = \left(1 - \frac{\pi}{16}\right) \frac{r^2}{kt} J_s \alpha_s = 0.804 \frac{r^2}{kt} J_s \alpha_s \qquad (56)$$

When the sun radiation is at x = $\pi r/2$, the problem is no longer symmetrical, and four equations of heat flow are required for the four quadrants of the tube. The heat flow equations are:

$$\frac{d^2T_1}{dx^2} = \frac{\sigma \epsilon T_m^4}{2kt} - \frac{J_s \alpha_s}{2kt} \sin\left(\frac{x}{r}\right) \qquad \left[0 < x < \frac{\pi r}{2}\right]$$

$$\frac{d^2T_2}{dx^2} = \frac{\sigma \epsilon T_m^4}{kt} - \frac{J_s \alpha_s}{kt} \sin\left(\frac{x}{r}\right) \qquad \left[\frac{\pi r}{2} < x < \pi r\right] \qquad (57)$$

$$\frac{d^2T_3}{dx^2} = \frac{\sigma \epsilon T_m^4}{kt} \qquad \left[\pi r < x < \frac{3\pi r}{2}\right]$$

$$\frac{d^2T_4}{dx^2} = \frac{\sigma \epsilon T_m^4}{2kt} \qquad \left[\frac{3\pi r}{2} < x < 2\pi r\right]$$

The solutions to equations (57) are:

$$T_1 = \frac{r^2}{kt} J_s \alpha_s \left[\frac{1}{4\pi}\left(\frac{x}{r}\right)^2 + \frac{1}{2}\sin\left(\frac{x}{r}\right) - 0.197\left(\frac{x}{r}\right) + 0.387 + C \right]$$

$$T_2 = \frac{r^2}{kt} J_s \alpha_s \left[\frac{1}{2\pi}\left(\frac{x}{r}\right)^2 + \sin\left(\frac{x}{r}\right) - 0.394\left(\frac{x}{r}\right) + C \right] \qquad (58)$$

$$T_3 = \frac{r^2}{kt} J_s \alpha_s \left[\frac{1}{2\pi}\left(\frac{x}{r} - \pi\right)^2 - 0.394\left(\frac{x}{r}\right) + 1.57 + C \right]$$

$$T_4 = \frac{r^2}{kt} J_s \alpha_s \left[\frac{1}{4\pi}\left(\frac{x}{r} - \pi\right)^2 - 0.197\left(\frac{x}{r}\right) + 0.84 + C \right]$$

By observation, it can be seen that the maximum temperature must occur in T_2, and the minimum in T_3. By differentation, the maximum temperature is found to occur at $x/r = 0.55\pi$ and the minimum at $x/r = 1.394\pi$. Using these values of x/r, the maximum temperature difference is:

$$T_2(x = 0.55\pi) - T_3(x = 1.394\pi) = \Delta T_{max} = 0.7 \frac{r^2}{kt} J_s \alpha_s \qquad (59)$$

Thus it can be seen that complete contact between layers in an overlapped tube decreases the maximum temperature difference across the tube by approximately 25%, for sun radiation only.

Further cases have not been considered because of the complexity of the equations involved, but it is felt that complete contact in the overlap will reduce temperature differences by the same order of magnitude for all cases.

PART III

LABORATORY TESTS

PART III

LABORATORY TESTS

1.0 INTRODUCTION

The purpose of this test is to measure the element tip deflection as a function of overlap, direction of incident radiation with respect to overlap, and temperature profile on the cross-section and along the length. The test apparatus may be used to investigate the effect of changes in tubular element design such as surface treatment, perforation, etc., giving orders of magnitude of improvements which may be expected in a space environment.

2.0 DESCRIPTION OF THE TEST APPARATUS

The layout of the test apparatus is shown in Figure 1 to 6. A double wall tube (1). 9 inches inside diameter and six feet high, encloses the upright tubular element (2) to be tested. This configuration eliminates the influence of weight as the expected tip deflection will be of the order of only 1/2 inch. The double-wall tube is watercooled and evacuated and is closed on the top by a glass plate (3) with a calibrated grid (4) to measure the tip deflection. A heater element (5) is mounted parallel to the tubular element in a watercooled housing (6). The length of the heater element is almost 5 feet and the capacity is 3 KW. The housing of the heater element is of such design that radiant energy is emitted normal to the tubular element through a 3/8 inch slot (7). The dissipated heat can be measured separately for each cooling device by measuring the water flow and temperature. The temperatures of the tubular element are measured by thermocouples soldered on the bottom, the middle and the top of the element, both on the bright and the dark side.

3.0 TEST LIMITATIONS

The test apparatus will not allow determination of absorptivity and reflectivity ratios for the tubular element with respect to sun light. This is because the ratios are obtained from the amount of solar radiation absorbed

or reflected in unit time by a unit area, integrated over the frequencies of the total sun light spectrum. In the test apparatus the radiation ratios will be different because the band of frequencies of the heater element is different and the frequencies are lower. However, comparature tests between materials and geometry are possible and will provide useful data. It can be safely assumed that any improvement in tube behavior which is noted in the test apparatus after a modification in tube design, will produce a similar improvement at least in direction and order of magnitude for the element in space.

4.0 TEST PROCEDURE

Preliminary tests showed that tip deflections were affected by the thermocouple leads soldered on the tubular elements. For this reason it was decided that two identical elements would be used for test purposes, one to allow temperature measurement with soldered thermocouples, the other to allow tip deflection measurements without the thermocouples attached.

It was intended that no test results would be tabulated until an accurate heat balance could be obtained, however, this was not possible in the time available. The best heat balance obtained showed an unaccounted for heat loss of 20%.

5.0 TEST RESULTS

5.1 Temperature Profile Along the Element Length

Temperature on the foot	560 to 570^{o}R
Temperature in the middle	586 to 591^{o}R
Temperature on the tip	584 to 589^{o}R

The temperatures are lower than they would be in space, which is to be expected due to the different heat source. Although sufficient time was allowed for the temperatures to settle, the gradients persisted and were observed in repeated tests. This suggests that the instrumentation or the heater input circuit must be improved.

5.2 Temperature Gradient Along the Perimeter

The maximum temperature differences between the bright and the dark side of the element were found to be as follows:

on the foot	9°F
in the middle	6.3°F
on the tip	5.4°F

These values also are lower than they would be in sun light. There should be no difference in the values on the foot and in the middle and on the tip. However, as the temperatures along the length differ it is understandable that a similar variation takes place for the temperatures around the perimeter.

5.3 Tip Deflection

The deflection was measured to be .35 inches. The theoretical deflection for a tubular element with the same temperature gradient around it's perimeter is .45 inches.

6.0 DISCUSSION OF THE TEST RESULTS

The limited test results indicate that the accuracy of the temperature measurements must be improved prior to further tests. It is believed that this can be achieved by selecting thermocouple materials and recording instruments that are more sensitive than those presently in use. The temperature of the heater element should be measured along its length to help explain the temperature gradient measured along the length of the tubular element.

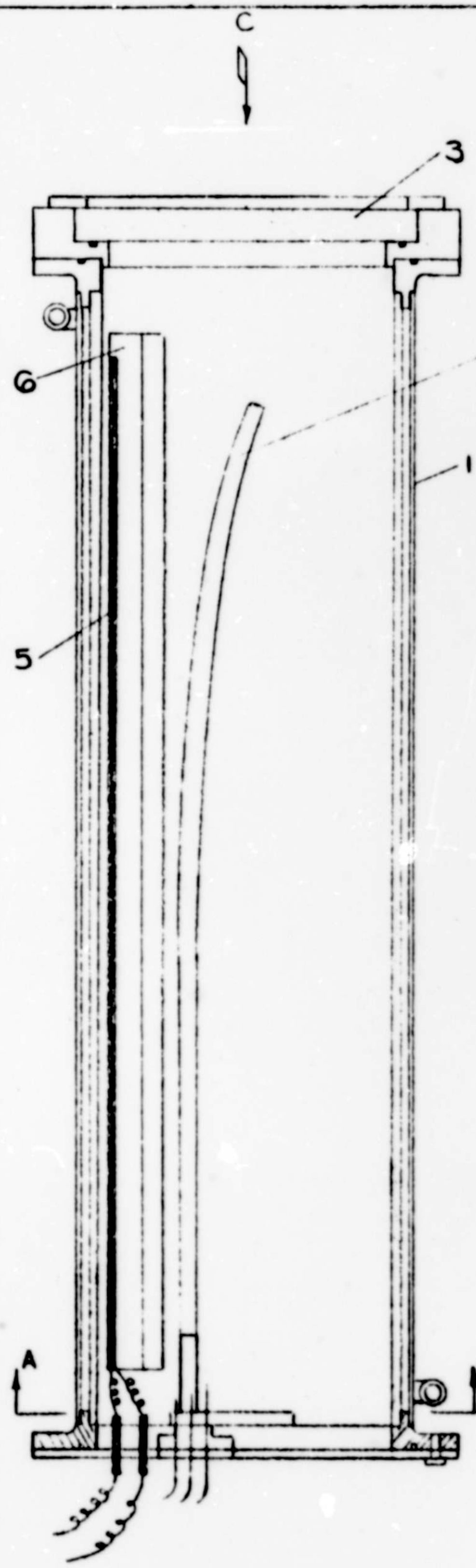
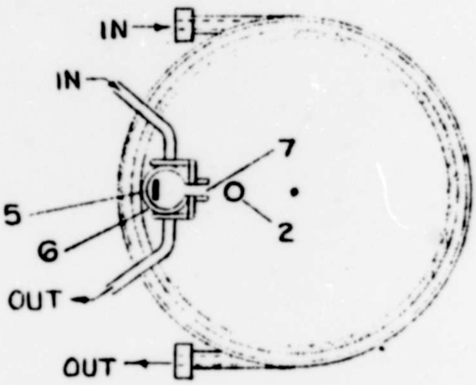
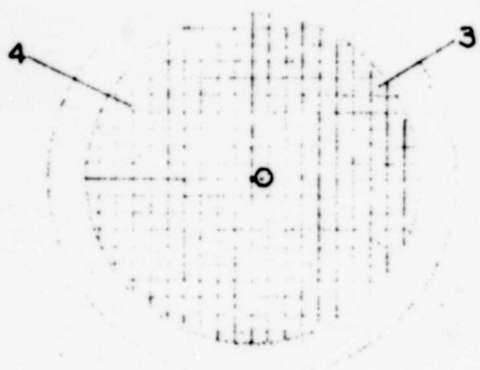


FIG 1



SECTION AA

FIG 2



VIEW ON ARROW 'C'

FIG 3.

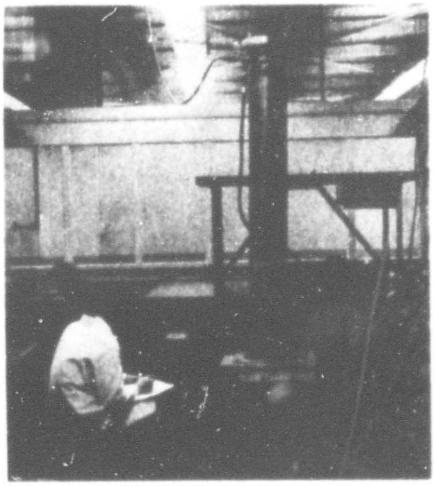


Figure 4. Reading Temperatures

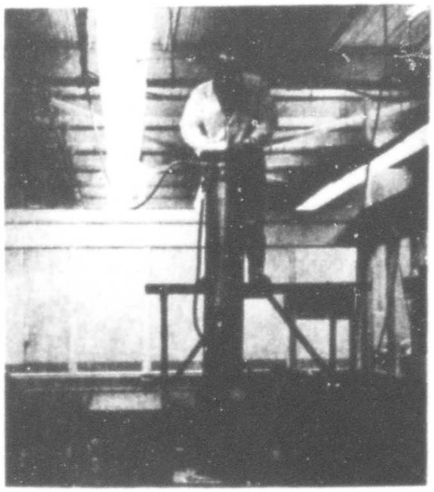


Figure 5. Measuring Tip Deflection

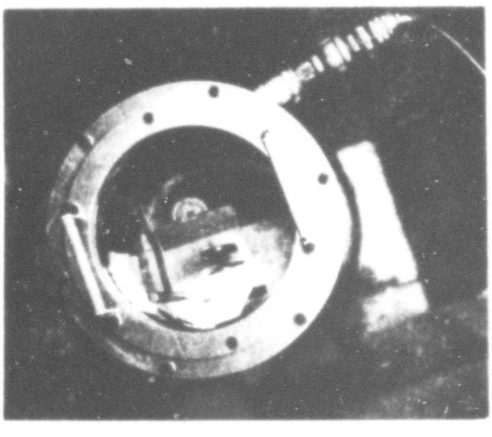


Figure 6. View Through the Glasscover

PART IV

CONCLUSIONS AND RECOMMENDATIONS

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CONCLUSIONS AND RECOMMENDATIONS

1.0 SCOPE OF THE MATHEMATICAL ANALYSIS

It is felt that the mathematical analysis has been developed sufficiently to provide a basis for predicting the distorted shapes of long tubular elements when exposed to radiation in space. Although test results to date have not completely verified the analysis, the equations can be used to predict the general shape and the orders of magnitude of the radius of curvature and tip deflection of tubular elements. The equations also show the influence of design par ameters such that the effects of design modifications maybe predicted quite accurately.

2.0 INFLUENCE OF DESIGN PARAMETERS

If it is desired to have tubular elements which remain "straight" when exposed to radiation in space, then it is necessary to choose design parameters which will decrease as much as possible the Profile Number

$$N = \frac{1}{4} \frac{de}{kt} J_s \alpha_s$$

so that the radius of curvature becomes very large according to the formula:

$$R = \frac{1}{N}$$

and the tip deflection becomes small according to the formula:

$$x = \frac{1}{2} N l^2$$

If the intensity of the incident radiation and the required diameter of the element are known, then the following parameters offer the possibility of either increasing or decreasing the Profile Number:

d/t = ratio of diameter to thickness
α_s = solar absorptivity
e = coefficient of thermal expansion
k = thermal conductivity

All four parameters are properties of the element material, since the value of the ratio d/t depends on the strength and fatigure properties of the metal used. Thus the problem of reducing element distortion in a space environment is simply a problem of choosing materials with a satisfactory combination of strength and thermal properties.

To illustrate the range of possibilities some figures may be given for differing materials.

The material of the specified tube is Beryllium Copper, with $e = 1.05 \times 10^{-6}$ and $N = 3 \times 10^{-3}$. A material such as Invar has a value of $e = 0.7 \times 10^{-6}$ which would reduce N to 0.21×10^{-3}, all other factors remaining constant. However, the thermal conductivity of Invar is smaller, and the absorptivity could be larger, than that for Beryllium Copper, giving a net reduction of N not quite so large as first anticipated.

The Profile Number N can also be reduced by decreasing the solar absorptivity α_s of the material. One method available is to silver coat the tube surface, which would decrease α_s from .45 to .05 and would also increase the overall thermal conductivity from 44 to 80 for a silver to Beryllium Copper thickness ratio of 1 to 4. This would reduce the Profile Number N from 3×10^{-3} to $.17 \times 10^{-3}$ assuming the factor d/t would remain constant.

One other method of reducing distortion not previously considered is to modify the design of the tube by perforating the tube material. By doing this, some of the radient energy would pass through the perforations on the bright side of the tube and be absorbed on the back side. In this way the temperature gradient around the perimeter would be reduced, thus causing smaller distortions.

3.0 RECOMMENDATIONS

Because of the relatively large deflections calculated for the specified Beryllium Copper tube, it is felt that considerable effort should be spent in verifying the theoretical calculations by extensive experimental work, and that further tests be carried out to determine the best possible means of overcoming these large deflections.

The verification of deflections calculated from the theoretical analysis can be achieved by use of test apparatus such as described in Part III of this report. Although no attempt has been made to duplicate sun radiation in this apparatus, it is sufficient to produce temperature gradients in the tube which can be related to the measured deflections. With some modifications to the apparatus and test procedure it is felt that the emissivity of the tube material can be determined, since the procedure represents a controlled steady-state heat transfer experiment.

In order to achieve smaller deflections it is suggested that a study of materials be made to determine which have the best combination of thermal and strength properties for reducing distortion. Except for solar absorptivity, the properties of most materials are well known and easily found. Thus the study would require extensive experimental work in determining solar absorptivities of materials which might be considered useful from a consideration of the other properties of interest.

Combinations of materials to use to best advantage the properties of each one should also be considered and studied. One example of this would be silver coating on a beryllium copper tubular element. The low solar absorptivity and high thermal conductivity of silver would combine well with the high strength properties of beryllium copper. Such a study would necessarily require information with regard to the best means of bonding the materials, optimum relative thickness of the materials and the best manufacturing and finishing methods available.

Methods for reducing distortion which primarily do not depend on the properties of the element materials should also be investigated. One such method would involve perforating the tubular element so that some of the solar radiation might pass through the hot side of the tube and be absorbed on the cold side, thus reducing temperatures gradients. Such a study would require an extensive mathematical analysis to determine patterns for the perforations and also to determine the effects of the perforations on radiation areas, heat conduction paths, and tube strength. The study would also require experimental tests to verify any theoretical calculations.

APPENDICES

LIST OF SYMBOLS

REFERENCES

LIST OF SYMBOLS

		units
A	Satellite altitude above earth surface	miles
B	Earth radiant energy factor (see 3.2, Part I)	....
F_A	Altitude correction factor (see 3.1, Part I)	....
J	Radiant energy	$BTU/hr.ft^2$
N	Element profile number (see 2.3, Part I)	ft^{-1}
R	Earth radius	miles
R_c	Radius of curvature	feet
T	Temperature	^oR
Index S,s	Refers to sun radiation	....
Index E,e	Refers to earth radiation	....
Index R,r	Refers to earth reflection	....
c	Specific heat	$Btu/lb\text{-}^oF$
d	Element diameter	feet
e	Thermal expansion coefficient	in/in
k	Thermal conductivity	$BTU/ft.\ ^oF\ hr$
l	Element length	feet
t	Element material thickness	feet
r	Element radius	feet
x,y,z	Axes of co-ordinate system	....
α	Absorptivity ratio	....

List of Symbols (cont'd)

γ	Reflectivity ratio	....
δ	Angle between sun and earth radiation direction with respect to the crossection of the element	degrees
ϵ	Emissivity ratio	....
η	Angle between perpendiculars from earth to satellite and earth to sun	degrees
θ	Radiation incident angle - angle between the element axis and the radiation direction seen from the root of the element.	degrees
ϕ	Overlap factor	....
ρ	Material density	lb/ft^3
σ	Stefan-Boltzmann constant	$BTU/ft^2 hr.(^oF)^4$
τ	Time	minutes
C_τ	Element time constant	minutes
q	Heat flow	BTU/hr.
E_b	Black body emissive power	$BTU/hr\text{-}ft^2$
C	Constant of integration	....

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3. BETZ, OLSON, SCHURIN, MORRIS: Determination of Emissivity and Reflectivity Data on Aircraft Structural Materials, Part II; WADC Technical Report 56-222, Armor Research Foundation, October, 1958.